

## SURFACE DEFORMATION DUE TO SHEAR AND TENSILE FAULTS IN A HALF-SPACE

BY YOSHIMITSU OKADA\*

### ABSTRACT

**A complete suite of closed analytical expressions is presented for the surface displacements, strains, and tilts due to inclined shear and tensile faults in a half-space for both point and finite rectangular sources. These expressions are particularly compact and free from field singular points which are inherent in the previously stated expressions of certain cases. The expressions derived here represent powerful tools not only for the analysis of static field changes associated with earthquake occurrence but also for the modeling of deformation fields arising from fluid-driven crack sources.**

### INTRODUCTION

Since dislocation theory was first introduced to the field of seismology by Steketee (1958), as well as a pioneer work by Rongved and Frasier (1958), numerous theoretical formulations describing the deformation of an isotropic homogeneous semi-infinite medium have been developed with increasing completeness and generality of source type and geometry. They range from the derivation of the surface displacement due to a point source of vertical strike-slip type in a Poisson solid (Steketee, 1958) to the strain fields at depth due to an inclined finite shear fault in a medium with arbitrary elastic constants (Iwasaki and Sato, 1979). The accomplishments of the various papers through which this progress has been achieved are summarized in Table 1.

Efforts to develop the formulations in a more realistic earth model have also been advanced through numerous studies, which include the effect of earth curvature (McGinley, 1969; Ben-Menahem *et al.*, 1969, 1970; Smylie and Mansinha, 1971), the effect of surface topography (Ishii and Takagi, 1967a; Takemoto, 1981; Segall and McTigue, 1984), the effect of crustal layering (Ishii and Takagi, 1967b; McGinley, 1969; Ben-Menahem and Gillon, 1970; Singh, 1970; Sato, 1971; Rybicki, 1971; Chinnery and Jovanovich, 1972; Sato and Matsu'ura, 1973; Jovanovich *et al.*, 1974a, b; Matsu'ura and Sato, 1975), the effect of lateral inhomogeneity (Rybicki, 1971, 1978; Rybicki and Kasahara, 1977; McHugh and Johnston, 1977; Niewiadomski and Rybicki, 1984), and the effect of obliquely layered medium (Sato, 1974; Sato and Yamashita, 1975). These studies revealed that the effect of earth curvature is negligible for the shallow events at distances of less than  $20^\circ$ , but that the vertical layering or lateral inhomogeneity can sometimes cause considerable effects on the deformation fields.

In spite of such an advance in calculating theoretical fields, the analyses of actual observations are still mostly based upon the simplest assumption of an isotropic homogeneous half-space and the simplest source configuration, largely for the following three reasons. First, it is most convenient and useful as the first approximation model. Second, the source model itself is inherently nonunique. Third, the quality of crustal movement data is generally poor at least up to the present

\* Present address: National Research Center for Disaster Prevention, Science and Technology Agency, Sakura-mura, Ibaraki, 305 Japan.

TABLE 1  
 PUBLISHED CLOSED ANALYTICAL EXPRESSIONS OF THE DEFORMATION DUE TO SHEAR AND TENSILE FAULTS IN A HALF-SPACE

POINT SOURCE	ORIENTATION		FAULT	MEDIUM		SURFACE DEFORMATION			DEFORMATION AT DEPTH							
	vertical	horiz inclined		$\lambda = \mu \lambda + \mu$	$U_x$	$U_y$	$U_z$	$\frac{\partial U_x}{\partial x}$	$\frac{\partial U_y}{\partial x}$	$\frac{\partial U_z}{\partial x}$	$\frac{\partial U_x}{\partial y}$	$\frac{\partial U_y}{\partial y}$	$\frac{\partial U_z}{\partial y}$	$\frac{\partial U_x}{\partial z}$	$\frac{\partial U_y}{\partial z}$	$\frac{\partial U_z}{\partial z}$
Steketee(1958)	○		○	○	○	○	○	○	○	○	○	○	○	○	○	○
Maruyama(1964)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Okada(1975)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Yamazaki(1978)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Iwasaki & Sato(1979)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
this study	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
<b>FINITE FAULT</b>																
Chinnery(1961)	○		○	○	○	○	○	○	○	○	○	○	○	○	○	○
Chinnery(1961, 1963)	○		○	○	○	○	○	○	○	○	○	○	○	○	○	○
Maruyama(1964)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Press(1965)	○		○	○	○	○	○	○	○	○	○	○	○	○	○	○
Savage & Hastie(1966)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Mansinha & Smylie(1967)	○		○	○	○	○	○	○	○	○	○	○	○	○	○	○
Mansinha & Smylie(1971)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Alewine(1974)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Sato & Matsu'ura(1974)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Yamazaki(1975)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Matsu'ura(1977)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Iwasaki & Sato(1979)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Matsu'ura & Tanioto(1980)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
Davis(1983)	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
this study	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

(Mikumo, 1973; Okada, 1980; Wyatt, 1982; Wyatt *et al.*, 1984). The last two factors often make it meaningless to compare the data with the predictions of an elaborate source or earth model.

The first objective of this paper is to check and review the closed analytical expressions which are already published to describe the surface deformation due to shear fault in a half-space. As our observations are restricted to near-ground surface, this class of solutions has the greatest practical importance to the study of the earthquake sources. Some of them, as presented, are too lengthy and complicated, while others have some singularities under the special conditions. For example, Savage and Hastie's (1966) formula is too complicated and cannot be applied to the vertical or horizontal fault, while Sato and Matsu'ura's (1974) formula results in "zero divide" at the points where the extensions of fault edges intersect the ground surface. Besides, misprints occur all too often in the published expressions. In this paper, the compact formulas to calculate the surface displacements, strains, and tilts due to a general shear fault in a half-space are given, which have been carefully checked to be free from any singularities.

The second objective of this paper is to add a heretofore unknown solution for the displacements, strains, and tilts arising from opening-mode dislocations. In contrast to the progress that has been made in the modeling of the deformation fields due to shear dislocations, the studies related to tensile fault are scarce as is seen in Table 1. The main reason for this is, no doubt, the importance that has been described to model the static field changes associated with earthquake occurrence. Tensile fault representation, which has a Burger's vector normal to the dislocation surface, also has some very important geophysical applications, such as a modeling of the deformation fields due to dyke injection in the volcanic region, mine collapse, or fluid-driven cracks.

Berry and Sales (1962) derived the surface displacement fields due to a closure of horizontal crack in a transversely isotropic medium. Maruyama (1964) gave the expressions of surface displacements due to vertical and horizontal tensile faults in a semi-infinite Poisson solid. Yamazaki (1978) treated the deformation fields arising from a dilatancy source. Davis (1983) derived an expression of the vertical displacement due to an inclined tensile fault in a half-space. He showed that this model can approximate well a tensile crack, just as shear dislocations are successfully used to approximate the deformation fields by shear cracks.

Recently, Evans and Wyatt (1984) found an interesting relation between changes in the water-head within a borehole and associated ground surface deformation in the surrounding region. Based upon the mechanism that subsurface hydraulically conductive fractures respond to changing fluid pressure, they suggested a quantitative tensile crack model to explain the observation. Their work has important implications for the measurement of crustal deformation in that it provides a physical basis for understanding an important class of crustal movement noise. It is well known that the precipitation is a major noise factor for crustal movement observation not only in the short period but also in the long one (Kasahara *et al.*, 1983), and it is definite that the precipitation affects the ground movement through some changes in the state of groundwater (Shichi and Okada, 1979; Edge *et al.*, 1981a, b; Takemoto, 1983). But so far, the effects of precipitation were mostly discussed with appropriate formal mathematical models (Takemoto, 1967; Tanaka, 1967; Sato *et al.*, 1980; Yanagisawa, 1980) or nonlinear tank model simulators

(Tanaka, 1979; Yamauchi, 1981). In the latter case, the water height in a certain tank is assumed to be proportional to the induced ground strain or tilt changes, being somewhat suggestive of Evans and Wyatt's (1984) model. The new solution presented here, for the surface deformation induced by an arbitrarily oriented rectangular opening-mode dislocation, provides a versatile and quantitative framework for evaluating and perhaps removing fluid-filled crack-induced noise from crustal deformation records.

### POINT SOURCE

Steketee (1958) showed that the displacement field  $u_i(x_1, x_2, x_3)$  due to a dislocation  $\Delta u_j(\xi_1, \xi_2, \xi_3)$  across a surface  $\Sigma$  in an isotropic medium is given by

$$u_i = \frac{1}{F} \int \int_{\Sigma} \Delta u_j \left[ \lambda \delta_{jk} \frac{\partial u_i^n}{\partial \xi_n} + \mu \left( \frac{\partial u_i^j}{\partial \xi_k} + \frac{\partial u_i^k}{\partial \xi_j} \right) \right] \nu_k d\Sigma \quad (1)$$

where,  $\delta_{jk}$  is the Kronecker delta,  $\lambda$  and  $\mu$  are Lamé's constants,  $\nu_k$  is the direction cosine of the normal to the surface element  $d\Sigma$ , and the summation convention

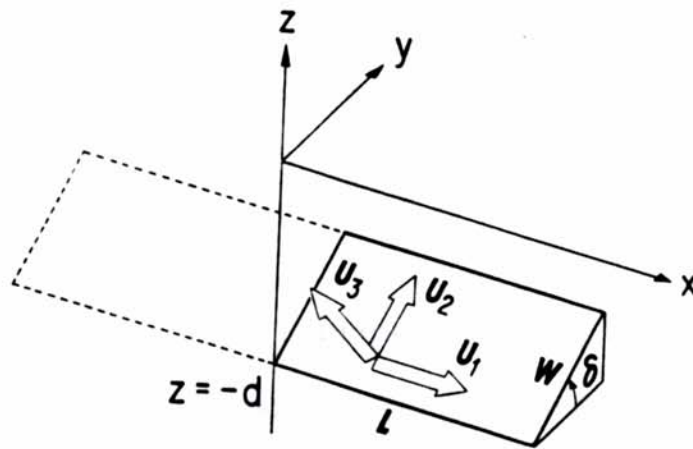


FIG. 1. Geometry of the source model.

applies.  $u_i^j$  is the  $i$ th component of the displacement at  $(x_1, x_2, x_3)$  due to the  $j$ th direction point force of magnitude  $F$  at  $(\xi_1, \xi_2, \xi_3)$ , whose expressions in a homogeneous half-space are listed in Press (1965).

Here, we take the Cartesian coordinate system as is shown in Figure 1. Elastic medium occupies the region of  $z \leq 0$  and  $x$  axis is taken to be parallel to the strike direction of the fault. Further, we define elementary dislocations  $U_1$ ,  $U_2$ , and  $U_3$  so as to correspond to strike-slip, dip-slip, and tensile components of arbitrary dislocation. In Figure 1, each vector represents the movement of hanging-wall side block relative to foot-wall side block. But note that, e.g., although  $U_2$  in Figure 1 shows reverse fault-type motion, this changes to normal fault-type movement if dip angle  $\delta$  becomes  $\sin 2\delta < 0$ . In this coordinate system,  $u_i^j$  at the ground surface are

expressed as follows

$$\left\{ \begin{aligned} u_1^1 &= \frac{F}{4\pi\mu} \left\{ \frac{1}{R} + \frac{(x_1 - \xi_1)^2}{R^3} + \frac{\mu}{\lambda + \mu} \left[ \frac{1}{R - \xi_3} - \frac{(x_1 - \xi_1)^2}{R(R - \xi_3)^2} \right] \right\} \\ u_2^1 &= \frac{F}{4\pi\mu} (x_1 - \xi_1)(x_2 - \xi_2) \left\{ \frac{1}{R^3} - \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)^2} \right\} \\ u_3^1 &= \frac{F}{4\pi\mu} (x_1 - \xi_1) \left\{ -\frac{\xi_3}{R^3} - \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)} \right\} \end{aligned} \right. \quad (2)$$

$$\left\{ \begin{aligned} u_1^2 &= \frac{F}{4\pi\mu} (x_1 - \xi_1)(x_2 - \xi_2) \left\{ \frac{1}{R^3} - \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)^2} \right\} \\ u_2^2 &= \frac{F}{4\pi\mu} \left\{ \frac{1}{R} + \frac{(x_2 - \xi_2)^2}{R^3} + \frac{\mu}{\lambda + \mu} \left[ \frac{1}{R - \xi_3} - \frac{(x_2 - \xi_2)^2}{R(R - \xi_3)^2} \right] \right\} \\ u_3^2 &= \frac{F}{4\pi\mu} (x_2 - \xi_2) \left\{ -\frac{\xi_3}{R^3} - \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)} \right\} \end{aligned} \right. \quad (3)$$

$$\left\{ \begin{aligned} u_1^3 &= \frac{F}{4\pi\mu} (x_1 - \xi_1) \left\{ -\frac{\xi_3}{R^3} + \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)} \right\} \\ u_2^3 &= \frac{F}{4\pi\mu} (x_2 - \xi_2) \left\{ -\frac{\xi_3}{R^3} + \frac{\mu}{\lambda + \mu} \frac{1}{R(R - \xi_3)} \right\} \\ u_3^3 &= \frac{F}{4\pi\mu} \left\{ \frac{1}{R} + \frac{\xi_3^2}{R^3} + \frac{\mu}{\lambda + \mu} \frac{1}{R} \right\} \end{aligned} \right. \quad (4)$$

where  $R^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + \xi_3^2$ .

Using equation (1), the contribution from surface element  $\Delta\Sigma$  of each elementary dislocation is written as follows

Strike-slip

$$\frac{1}{F} \mu U_1 \Delta\Sigma \left[ -\left( \frac{\partial u_i^1}{\partial \xi_2} + \frac{\partial u_i^2}{\partial \xi_1} \right) \sin \delta + \left( \frac{\partial u_i^1}{\partial \xi_3} + \frac{\partial u_i^3}{\partial \xi_1} \right) \cos \delta \right]. \quad (5)$$

Dip-slip

$$\frac{1}{F} \mu U_2 \Delta\Sigma \left[ \left( \frac{\partial u_i^2}{\partial \xi_3} + \frac{\partial u_i^3}{\partial \xi_2} \right) \cos 2\delta + \left( \frac{\partial u_i^3}{\partial \xi_3} - \frac{\partial u_i^2}{\partial \xi_2} \right) \sin 2\delta \right]. \quad (6)$$

## Tensile

$$\frac{1}{F} U_3 \Delta \Sigma \left[ \lambda \frac{\partial u_i^n}{\partial \xi_n} + 2\mu \left( \frac{\partial u_i^2}{\partial \xi_2} \sin^2 \delta + \frac{\partial u_i^3}{\partial \xi_3} \cos^2 \delta \right) - \mu \left( \frac{\partial u_i^2}{\partial \xi_3} + \frac{\partial u_i^3}{\partial \xi_2} \right) \sin 2\delta \right]. \quad (7)$$

These show the body force equivalents of a double-couple with moment  $\mu U_1 \Delta \Sigma$  or  $\mu U_2 \Delta \Sigma$  in case of shear fault, and a center of dilatation (intensity  $\lambda U_3 \Delta \Sigma$ ) combined with a double-couple without moment (intensity  $2\mu U_3 \Delta \Sigma$ ) in case of tensile fault. Substituting (2), (3), and (4) into (5), (6), and (7), and setting  $\xi_1 = \xi_2 = 0$ ,  $\xi_3 = -d$ , we can get the surface displacements due to a point source located at  $(0, 0, -d)$ , from which the surface strains and tilts can be easily obtained by differentiation. The final results are listed below using  $(x, y, z)$  instead of  $(x_1, x_2, x_3)$  and the superscript  $^0$  to distinguish the quantities related to point sources.

## (1) Displacements

## For strike-slip

$$\begin{cases} u_x^0 = -\frac{U_1}{2\pi} \left[ \frac{3x^2q}{R^5} + I_1^0 \sin \delta \right] \Delta \Sigma \\ u_y^0 = -\frac{U_1}{2\pi} \left[ \frac{3xyq}{R^5} + I_2^0 \sin \delta \right] \Delta \Sigma \\ u_z^0 = -\frac{U_1}{2\pi} \left[ \frac{3xdq}{R^5} + I_4^0 \sin \delta \right] \Delta \Sigma. \end{cases} \quad (8)$$

## For dip-slip

$$\begin{cases} u_x^0 = -\frac{U_2}{2\pi} \left[ \frac{3xpq}{R^5} - I_3^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ u_y^0 = -\frac{U_2}{2\pi} \left[ \frac{3ypq}{R^5} - I_1^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ u_z^0 = -\frac{U_2}{2\pi} \left[ \frac{3dpq}{R^5} - I_5^0 \sin \delta \cos \delta \right] \Delta \Sigma. \end{cases} \quad (9)$$

## For tensile fault

$$\begin{cases} u_x^0 = \frac{U_3}{2\pi} \left[ \frac{3xq^2}{R^5} - I_3^0 \sin^2 \delta \right] \Delta \Sigma \\ u_y^0 = \frac{U_3}{2\pi} \left[ \frac{3yq^2}{R^5} - I_1^0 \sin^2 \delta \right] \Delta \Sigma \\ u_z^0 = \frac{U_3}{2\pi} \left[ \frac{3dq^2}{R^5} - I_5^0 \sin^2 \delta \right] \Delta \Sigma \end{cases} \quad (10)$$

where

$$\begin{cases} I_1^0 = \frac{\mu}{\lambda + \mu} y \left[ \frac{1}{R(R+d)^2} - x^2 \frac{3R+d}{R^3(R+d)^3} \right] \\ I_2^0 = \frac{\mu}{\lambda + \mu} x \left[ \frac{1}{R(R+d)^2} - y^2 \frac{3R+d}{R^3(R+d)^3} \right] \\ I_3^0 = \frac{\mu}{\lambda + \mu} \left[ \frac{x}{R^3} \right] - I_2^0 \\ I_4^0 = \frac{\mu}{\lambda + \mu} \left[ -xy \frac{2R+d}{R^3(R+d)^2} \right] \\ I_5^0 = \frac{\mu}{\lambda + \mu} \left[ \frac{1}{R(R+d)} - x^2 \frac{2R+d}{R^3(R+d)^2} \right] \end{cases} \quad (11)$$

$$\begin{cases} p = y \cos \delta + d \sin \delta \\ q = y \sin \delta - d \cos \delta \\ R^2 = x^2 + y^2 + d^2 = x^2 + p^2 + q^2. \end{cases} \quad (12)$$

## (2) Strains

For strike-slip

$$\begin{cases} \frac{\partial u_x^0}{\partial x} = -\frac{U_1}{2\pi} \left[ \frac{3xq}{R^5} \left( 2 - \frac{5x^2}{R^2} \right) + J_1^0 \sin \delta \right] \Delta \Sigma \\ \frac{\partial u_x^0}{\partial y} = -\frac{U_1}{2\pi} \left[ -\frac{15x^2 y q}{R^7} + \left( \frac{3x^2}{R^5} + J_2^0 \right) \sin \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial x} = -\frac{U_1}{2\pi} \left[ \frac{3yq}{R^5} \left( 1 - \frac{5x^2}{R^2} \right) + J_2^0 \sin \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial y} = -\frac{U_1}{2\pi} \left[ \frac{3xq}{R^5} \left( 1 - \frac{5y^2}{R^2} \right) + \left( \frac{3xy}{R^5} + J_4^0 \right) \sin \delta \right] \Delta \Sigma. \end{cases} \quad (13)$$

For dip-slip

$$\begin{cases} \frac{\partial u_x^0}{\partial x} = -\frac{U_2}{2\pi} \left[ \frac{3pq}{R^5} \left( 1 - \frac{5x^2}{R^2} \right) - J_3^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ \frac{\partial u_x^0}{\partial y} = -\frac{U_2}{2\pi} \left[ \frac{3x}{R^5} \left( s - \frac{5ypq}{R^2} \right) - J_1^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial x} = -\frac{U_2}{2\pi} \left[ -\frac{15xypq}{R^7} - J_1^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial y} = -\frac{U_2}{2\pi} \left[ \frac{3pq}{R^5} \left( 1 - \frac{5y^2}{R^2} \right) + \frac{3ys}{R^5} - J_2^0 \sin \delta \cos \delta \right] \Delta \Sigma. \end{cases} \quad (14)$$

For tensile fault

$$\left\{ \begin{aligned} \frac{\partial u_x^0}{\partial x} &= \frac{U_3}{2\pi} \left[ \frac{3q^2}{R^5} \left( 1 - \frac{5x^2}{R^2} \right) - J_3^0 \sin^2 \delta \right] \Delta \Sigma \\ \frac{\partial u_x^0}{\partial y} &= \frac{U_3}{2\pi} \left[ \frac{3xq}{R^5} \left( 2 \sin \delta - \frac{5yq}{R^2} \right) - J_1^0 \sin^2 \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial x} &= \frac{U_3}{2\pi} \left[ -\frac{15xyq^2}{R^7} - J_1^0 \sin^2 \delta \right] \Delta \Sigma \\ \frac{\partial u_y^0}{\partial y} &= \frac{U_3}{2\pi} \left[ \frac{3q}{R^5} \left( q + 2y \sin \delta - \frac{5y^2q}{R^2} \right) - J_2^0 \sin^2 \delta \right] \Delta \Sigma \end{aligned} \right. \quad (15)$$

where  $s = p \sin \delta + q \cos \delta$  and

$$\left\{ \begin{aligned} J_1^0 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{3R + d}{R^3(R + d)^3} + 3x^3y \frac{5R^2 + 4Rd + d^2}{R^5(R + d)^4} \right] \\ J_2^0 &= \frac{\mu}{\lambda + \mu} \left[ \frac{1}{R^3} - \frac{3}{R(R + d)^2} + 3x^2y^2 \frac{5R^2 + 4Rd + d^2}{R^5(R + d)^4} \right] \\ J_3^0 &= \frac{\mu}{\lambda + \mu} \left[ \frac{1}{R^3} - \frac{3x^2}{R^5} \right] - J_2^0 \\ J_4^0 &= \frac{\mu}{\lambda + \mu} \left[ -\frac{3xy}{R^5} \right] - J_1^0. \end{aligned} \right. \quad (16)$$

### (3) Tilts

For strike-slip

$$\left\{ \begin{aligned} \frac{\partial u_z^0}{\partial x} &= -\frac{U_1}{2\pi} \left[ \frac{3dq}{R^5} \left( 1 - \frac{5x^2}{R^2} \right) + K_1^0 \sin \delta \right] \Delta \Sigma \\ \frac{\partial u_z^0}{\partial y} &= -\frac{U_1}{2\pi} \left[ -\frac{15xydq}{R^7} + \left( \frac{3xd}{R^5} + K_2^0 \right) \sin \delta \right] \Delta \Sigma. \end{aligned} \right. \quad (17)$$

For dip-slip

$$\left\{ \begin{aligned} \frac{\partial u_z^0}{\partial x} &= -\frac{U_2}{2\pi} \left[ -\frac{15xdpq}{R^7} - K_3^0 \sin \delta \cos \delta \right] \Delta \Sigma \\ \frac{\partial u_z^0}{\partial y} &= -\frac{U_2}{2\pi} \left[ \frac{3d}{R^5} \left( s - \frac{5ypq}{R^2} \right) - K_1^0 \sin \delta \cos \delta \right] \Delta \Sigma. \end{aligned} \right. \quad (18)$$



For tensile fault

$$\begin{cases} \frac{\partial u_z^0}{\partial x} = \frac{U_3}{2\pi} \left[ -\frac{15xdq^2}{R^7} - K_3^0 \sin^2 \delta \right] \Delta\Sigma \\ \frac{\partial u_z^0}{\partial y} = \frac{U_3}{2\pi} \left[ \frac{3dq}{R^5} \left( 2\sin \delta - \frac{5yq}{R^2} \right) - K_1^0 \sin^2 \delta \right] \Delta\Sigma \end{cases} \quad (19)$$

where

$$\begin{cases} K_1^0 = -\frac{\mu}{\lambda + \mu} y \left[ \frac{2R + d}{R^3(R + d)^2} - x^2 \frac{8R^2 + 9Rd + 3d^2}{R^5(R + d)^3} \right] \\ K_2^0 = -\frac{\mu}{\lambda + \mu} x \left[ \frac{2R + d}{R^3(R + d)^2} - y^2 \frac{8R^2 + 9Rd + 3d^2}{R^5(R + d)^3} \right] \\ K_3^0 = -\frac{\mu}{\lambda + \mu} \left[ \frac{3xd}{R^5} \right] - K_2^0. \end{cases} \quad (20)$$

#### FINITE RECTANGULAR SOURCE

For a finite rectangular fault with length  $L$  and width  $W$  (Figure 1), the deformation field can be derived by taking  $x - \xi'$ ,  $y - \eta' \cos \delta$  and  $d - \eta' \sin \delta$  in place of  $x$ ,  $y$ , and  $d$  in the equations obtained in the previous section and by performing the integration

$$\int_0^L d\xi' \int_0^W d\eta'. \quad (21)$$

Following Sato and Matsu'ura (1974), it is convenient to change variables from  $\xi'$ ,  $\eta'$  to  $\xi$ ,  $\eta$  by

$$\begin{cases} x - \xi' = \xi \\ p - \eta' = \eta \end{cases} \quad (22)$$

where,  $p = y \cos \delta + d \sin \delta$  as before. Then, the above integration becomes

$$\int_x^{x-L} d\xi \int_p^{p-W} d\eta. \quad (23)$$

The final results condensed into compact forms are listed below using Chinnery's notation  $\parallel$  to represent the substitution

$$f(\xi, \eta) \parallel = f(x, p) - f(x, p - W) - f(x - L, p) + f(x - L, p - W). \quad (24)$$

If we take a rectangular fault with length  $2L$  (dashed line in Figure 1), it is only necessary to replace  $x$  in the first and the second terms of the right-hand side of equation (24) to  $x + L$ .

## (1) Displacements

For strike-slip

$$\begin{cases} u_x = -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R(R+\eta)} + \tan^{-1} \frac{\xi \eta}{qR} + I_1 \sin \delta \right] \\ u_y = -\frac{U_1}{2\pi} \left[ \frac{\hat{y} q}{R(R+\eta)} + \frac{q \cos \delta}{R+\eta} + I_2 \sin \delta \right] \\ u_z = -\frac{U_1}{2\pi} \left[ \frac{\hat{d} q}{R(R+\eta)} + \frac{q \sin \delta}{R+\eta} + I_4 \sin \delta \right] \end{cases} \quad (25)$$

For dip-slip

$$\begin{cases} u_x = -\frac{U_2}{2\pi} \left[ \frac{q}{R} - I_3 \sin \delta \cos \delta \right] \\ u_y = -\frac{U_2}{2\pi} \left[ \frac{\hat{y} q}{R(R+\xi)} + \cos \delta \tan^{-1} \frac{\xi \eta}{qR} - I_1 \sin \delta \cos \delta \right] \\ u_z = -\frac{U_2}{2\pi} \left[ \frac{\hat{d} q}{R(R+\xi)} + \sin \delta \tan^{-1} \frac{\xi \eta}{qR} - I_5 \sin \delta \cos \delta \right] \end{cases} \quad (26)$$

For tensile fault

$$\begin{cases} u_x = \frac{U_3}{2\pi} \left[ \frac{q^2}{R(R+\eta)} - I_3 \sin^2 \delta \right] \\ u_y = \frac{U_3}{2\pi} \left[ \frac{-\hat{d} q}{R(R+\xi)} - \sin \delta \left\{ \frac{\xi q}{R(R+\eta)} - \tan^{-1} \frac{\xi \eta}{qR} \right\} - I_1 \sin^2 \delta \right] \\ u_z = \frac{U_3}{2\pi} \left[ \frac{\hat{y} q}{R(R+\xi)} + \cos \delta \left\{ \frac{\xi q}{R(R+\eta)} - \tan^{-1} \frac{\xi \eta}{qR} \right\} - I_5 \sin^2 \delta \right] \end{cases} \quad (27)$$

where

$$\begin{cases} I_1 = \frac{\mu}{\lambda + \mu} \left[ \frac{-1}{\cos \delta} \frac{\xi}{R + \hat{d}} \right] - \frac{\sin \delta}{\cos \delta} I_5 \\ I_2 = \frac{\mu}{\lambda + \mu} [-\ln(R + \eta)] - I_3 \\ I_3 = \frac{\mu}{\lambda + \mu} \left[ \frac{1}{\cos \delta} \frac{\hat{y}}{R + \hat{d}} - \ln(R + \eta) \right] + \frac{\sin \delta}{\cos \delta} I_4 \\ I_4 = \frac{\mu}{\lambda + \mu} \frac{1}{\cos \delta} [\ln(R + \hat{d}) - \sin \delta \ln(R + \eta)] \\ I_5 = \frac{\mu}{\lambda + \mu} \frac{2}{\cos \delta} \tan^{-1} \frac{\eta(X + q \cos \delta) + X(R + X) \sin \delta}{\xi(R + X) \cos \delta} \end{cases} \quad (28)$$

and if  $\cos \delta = 0$ ,

$$\begin{cases} I_1 = -\frac{\mu}{2(\lambda + \mu)} \frac{\xi q}{(R + \bar{d})^2} \\ I_3 = \frac{\mu}{2(\lambda + \mu)} \left[ \frac{\eta}{R + \bar{d}} + \frac{\dot{y}q}{(R + \bar{d})^2} - \ln(R + \eta) \right] \\ I_4 = -\frac{\mu}{\lambda + \mu} \frac{q}{R + \bar{d}} \\ I_5 = -\frac{\mu}{\lambda + \mu} \frac{\xi \sin \delta}{R + \bar{d}} \end{cases} \quad (29)$$

$$\begin{cases} p = y \cos \delta + d \sin \delta \\ q = y \sin \delta - d \cos \delta \\ \bar{y} = \eta \cos \delta + q \sin \delta \\ \bar{d} = \eta \sin \delta - q \cos \delta \\ R^2 = \xi^2 + \eta^2 + q^2 = \xi^2 + \bar{y}^2 + \bar{d}^2 \\ X^2 = \xi^2 + q^2. \end{cases} \quad (30)$$

When  $\cos \delta = 0$ , we must be careful that there are two cases of  $\sin \delta = +1$  and  $-1$ .

(2) Strains

For strike-slip

$$\begin{cases} \frac{\partial u_x^0}{\partial x} = \frac{U_1}{2\pi} \left[ \xi^2 q A_n - J_1 \sin \delta \right] \parallel \\ \frac{\partial u_x^0}{\partial y} = \frac{U_1}{2\pi} \left[ \frac{\xi^3 \bar{d}}{R^3 (\eta^2 + q^2)} - (\xi^3 A_n + J_2) \sin \delta \right] \parallel \\ \frac{\partial u_y^0}{\partial x} = \frac{U_1}{2\pi} \left[ \frac{\xi q}{R^3} \cos \delta + (\xi q^2 A_n - J_2) \sin \delta \right] \parallel \\ \frac{\partial u_y^2}{\partial y} = \frac{U_1}{2\pi} \left[ \frac{\dot{y}q}{R^3} \cos \delta + \left\{ q^3 A_n \sin \delta - \frac{2q \sin \delta}{R(R + \eta)} - \frac{\xi^2 + \eta^2}{R^3} \cos \delta - J_4 \right\} \sin \delta \right] \parallel \end{cases} \quad (31)$$

For dip-slip

$$\left\{ \begin{array}{l} \frac{\partial u_x^0}{\partial x} = \frac{U_2}{2\pi} \left[ \frac{\xi q}{R^3} + J_3 \sin \delta \cos \delta \right] \parallel \\ \frac{\partial u_x^0}{\partial y} = \frac{U_2}{2\pi} \left[ \frac{\tilde{y} q}{R^3} - \frac{\sin \delta}{R} + J_1 \sin \delta \cos \delta \right] \parallel \\ \frac{\partial u_y^0}{\partial x} = \frac{U_2}{2\pi} \left[ \frac{\tilde{y} q}{R^3} + \frac{q \cos \delta}{R(R + \eta)} + J_1 \sin \delta \cos \delta \right] \parallel \\ \frac{\partial u_y^0}{\partial y} = \frac{U_2}{2\pi} \left[ \tilde{y}^2 q A_\xi - \left\{ \frac{2\tilde{y}}{R(R + \xi)} + \frac{\xi \cos \delta}{R(R + \eta)} \right\} \sin \delta + J_2 \sin \delta \cos \delta \right] \parallel \end{array} \right. \quad (32)$$

For tensile fault

$$\left\{ \begin{array}{l} \frac{\partial u_x}{\partial x} = -\frac{U_3}{2\pi} \left[ \xi q^2 A_\eta + J_3 \sin^2 \delta \right] \parallel \\ \frac{\partial u_x}{\partial y} = -\frac{U_3}{2\pi} \left[ -\frac{\tilde{d} q}{R^3} - \xi^2 q A_\eta \sin \delta + J_1 \sin^2 \delta \right] \parallel \\ \frac{\partial u_y}{\partial x} = -\frac{U_3}{2\pi} \left[ \frac{q^2}{R^3} \cos \delta + q^3 A_\eta \sin \delta + J_1 \sin^2 \delta \right] \parallel \\ \frac{\partial u_y}{\partial y} = -\frac{U_3}{2\pi} \left[ (\tilde{y} \cos \delta - \tilde{d} \sin \delta) q^2 A_\xi - \frac{q \sin 2\delta}{R(R + \xi)} - (\xi q^2 A_\eta - J_2) \sin^2 \delta \right] \parallel \end{array} \right. \quad (33)$$

where

$$\left\{ \begin{array}{l} J_1 = \frac{\mu}{\lambda + \mu} \frac{1}{\cos \delta} \left[ \frac{\xi^2}{R(R + \tilde{d})^2} - \frac{1}{R + \tilde{d}} \right] - \frac{\sin \delta}{\cos \delta} K_3 \\ J_2 = \frac{\mu}{\lambda + \mu} \frac{1}{\cos \delta} \left[ \frac{\xi \tilde{y}}{R(R + \tilde{d})^2} \right] - \frac{\sin \delta}{\cos \delta} K_1 \\ J_3 = \frac{\mu}{\lambda + \mu} \left[ -\frac{\xi}{R(R + \eta)} \right] - J_2 \\ J_4 = \frac{\mu}{\lambda + \mu} \left[ -\frac{\cos \delta}{R} - \frac{q \sin \delta}{R(R + \eta)} \right] - J_1 \end{array} \right. \quad (34)$$

$K_1$  and  $K_3$  are given in equation (40), and if  $\cos \delta = 0$ ,

$$\begin{cases} J_1 = \frac{\mu}{2(\lambda + \mu)} \frac{q}{(R + \bar{d})^2} \left[ \frac{2\xi^2}{R(R + \bar{d})} - 1 \right] \\ J_2 = \frac{\mu}{2(\lambda + \mu)} \frac{\xi \sin \delta}{(R + \bar{d})^2} \left[ \frac{2q^2}{R(R + \bar{d})} - 1 \right] \end{cases} \quad (35)$$

$$\begin{cases} A_\xi = \frac{2R + \xi}{R^3(R + \xi)^2} \\ A_\eta = \frac{2R + \eta}{R^3(R + \eta)^2} \end{cases} \quad (36)$$

(3) Tilts

For strike-slip

$$\begin{cases} \frac{\partial u_z}{\partial x} = \frac{U_1}{2\pi} \left[ -\xi q^2 A_\eta \cos \delta + \left( \frac{\xi q}{R^3} - K_1 \right) \sin \delta \right] \parallel \\ \frac{\partial u_z}{\partial y} = \frac{U_1}{2\pi} \left[ \frac{\bar{d}q}{R^3} \cos \delta + \left( \xi^2 q A_\eta \cos \delta - \frac{\sin \delta}{R} + \frac{\dot{y}q}{R^3} - K_2 \right) \sin \delta \right] \parallel \end{cases} \quad (37)$$

For dip-slip

$$\begin{cases} \frac{\partial u_z}{\partial x} = \frac{U_2}{2\pi} \left[ \frac{\bar{d}q}{R^3} + \frac{q \sin \delta}{R(R + \eta)} + K_3 \sin \delta \cos \delta \right] \parallel \\ \frac{\partial u_z}{\partial y} = \frac{U_2}{2\pi} \left[ \dot{y} \bar{d} q A_\xi - \left\{ \frac{2\bar{d}}{R(R + \xi)} + \frac{\xi \sin \delta}{R(R + \eta)} \right\} \sin \delta + K_1 \sin \delta \cos \delta \right] \parallel \end{cases} \quad (38)$$

For tensile fault

$$\begin{cases} \frac{\partial u_z}{\partial x} = -\frac{U_3}{2\pi} \left[ \frac{q^2}{R^3} \sin \delta - q^3 A_\eta \cos \delta + K_3 \sin^2 \delta \right] \parallel \\ \frac{\partial u_z}{\partial y} = -\frac{U_3}{2\pi} \left[ (\dot{y} \sin \delta + \bar{d} \cos \delta) q^2 A_\xi + \xi q^2 A_\eta \sin \delta \cos \delta \right. \\ \left. - \left\{ \frac{2q}{R(R + \xi)} - K_1 \right\} \sin^2 \delta \right] \parallel \end{cases} \quad (39)$$

where

$$\begin{cases} K_1 = \frac{\mu}{\lambda + \mu} \frac{\xi}{\cos \delta} \left[ \frac{1}{R(R + \bar{d})} - \frac{\sin \delta}{R(R + \eta)} \right] \\ K_2 = \frac{\mu}{\lambda + \mu} \left[ -\frac{\sin \delta}{R} + \frac{q \cos \delta}{R(R + \eta)} \right] - K_3 \\ K_3 = \frac{\mu}{\lambda + \mu} \frac{1}{\cos \delta} \left[ \frac{q}{R(R + \eta)} - \frac{\bar{y}}{R(R + \bar{d})} \right] \end{cases} \quad (40)$$

and if  $\cos \delta = 0$ ,

$$\begin{cases} K_1 = \frac{\mu}{\lambda + \mu} \frac{\xi q}{R(R + \bar{d})^2} \\ K_3 = \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{R + \bar{d}} \left[ \frac{\xi^2}{R(R + \bar{d})} - 1 \right] \end{cases} \quad (41)$$

In the indefinite integral expressions stated in this section, some terms become singular at the special conditions. Returning to the integral (23) and carefully checking these special cases, we can reach the following rules to avoid all the singularities. (i) When  $q = 0$ , set  $\tan^{-1}(\xi\eta/qR) = 0$  in equations (25) to (27). (ii) When  $\xi = 0$ , set  $I_5 = 0$  in equation (28). (iii) When  $R + \eta = 0$  (this occurs only when  $\sin \delta < 0$  and  $\xi = q = 0$ ), set all the terms which contain  $R + \eta$  in their denominators to be zero in equations (25) to (40), and replace  $\ln(R + \eta)$  to  $-\ln(R - \eta)$  in equations (28) and (29).

#### DISCUSSION

A compact analytical expression of the surface displacements, strains, and tilts due to inclined shear and tensile faults in a half-space are given for both point and finite rectangular sources in the preceding sections. All similar expressions known to the author were checked to be equivalent to the formulas given here except for some misprints in the literatures, which are now listed in the Appendix.

The formulas for point sources derived here can be used as an alternative of Maruyama's (1964) expressions to estimate far-field deformation or to construct the deformation fields by more general faults. The formulas for finite shear fault derived here are essentially identical to those of Matsu'ura (1977) as to the displacements and Sato and Matsu'ura (1974) as to the strains and tilts. But here, some revisions have been made to overcome the following difficulties which are included in the previous expressions. (i) On the line where the extension of the fault plane intersects the ground surface, the displacement becomes singular. (ii) On the lines where the vertical planes containing the inclined edges of the fault intersect the ground surface, the vertical displacement becomes singular. (iii) Displacements cannot be evaluated in case of  $\delta = -\pi/2$ . (iv) At the points where the inclined edges of the fault intersect the ground surface, the strains and tilts become singular. In addition to this revision, the formulas for tensile fault are newly added in this paper, and the work to derive the expressions of the surface deformation fields due to buried rectangular faults in a half-space seems to have come to maturity now.

TABLE 2  
CHECKLIST FOR NUMERICAL CALCULATIONS

	$u_x$	$u_y$	$u_z$	$\frac{du_x}{dx}$	$\frac{du_x}{dy}$	$\frac{du_x}{dz}$	$\frac{du_y}{dx}$	$\frac{du_y}{dy}$	$\frac{du_y}{dz}$	$\frac{du_z}{dx}$	$\frac{du_z}{dy}$	$\frac{du_z}{dz}$
Case 1 <sup>a</sup>												
Strike	-9.447E-4	-1.023E-3	-7.420E-4	-2.286E-4	-1.425E-4	-2.051E-4	-2.007E-4	-3.007E-4	-6.259E-5	-1.693E-4	-6.345E-4	+8.104E-4
Dip	-1.172E-3	-2.082E-3	-2.532E-3	-1.526E-4	-3.544E-4	+6.983E-4	+6.811E-4	-1.154E-3	+8.707E-4	-6.345E-4	+8.104E-4	+8.104E-4
Tensile	-3.572E-4	+3.531E-4	-2.007E-4	-1.360E-4	+5.073E-4	-6.773E-5	+6.811E-4	+6.811E-4	+7.541E-5	+8.104E-4	+8.104E-4	+8.104E-4
Case 2 <sup>b</sup>												
Strike	-8.689E-3	-4.298E-3	-2.747E-3	-1.220E-3	+2.470E-4	-8.191E-3	-5.814E-4	-5.814E-4	-5.175E-3	+2.945E-4	+2.945E-4	+2.945E-4
Dip	-4.682E-3	-3.527E-2	-3.564E-2	-8.867E-3	-1.519E-4	+4.057E-3	-1.035E-2	-1.035E-2	+4.088E-3	+2.626E-3	+2.626E-3	+2.626E-3
Tensile	-2.660E-4	+1.056E-2	+3.214E-3	-5.655E-4	+1.993E-3	-1.066E-3	+1.230E-2	+1.230E-2	-3.730E-4	+1.040E-2	+1.040E-2	+1.040E-2
Case 3 <sup>c</sup>												
Strike	0	+5.253E-3	0	0	-1.864E-2	-2.325E-3	0	0	0	+2.289E-2	+2.289E-2	+2.289E-2
Dip	0	0	0	0	+2.748E-2	0	0	0	0	-7.166E-2	-7.166E-2	-7.166E-2
Tensile	+1.223E-2	0	-1.606E-2	-4.182E-3	0	0	-2.325E-3	-2.325E-3	-9.146E-3	0	0	0
Case 4 <sup>d</sup>												
Strike	0	-1.303E-3	0	0	+2.726E-3	+7.345E-4	0	0	0	-4.422E-3	-4.422E-3	-4.422E-3
Dip	0	0	0	0	+5.157E-3	0	0	0	0	-1.901E-2	-1.901E-2	-1.901E-2
Tensile	+3.507E-3	0	-7.740E-3	-1.770E-3	0	0	-7.345E-4	-7.345E-4	-1.843E-3	0	0	0

<sup>a</sup> Point source:  $x = 2; y = 3; d = 4; \delta = 70^\circ$ .

<sup>b</sup> Finite source:  $x = 2; y = 3; d = 4; \delta = 70^\circ; L = 3; W = 2$ .

<sup>c</sup> Finite source:  $x = 0; y = 0; d = 4; \delta = 90^\circ; L = 3; W = 2$ .

<sup>d</sup> Finite source:  $x = 0; y = 0; d = 4; \delta = -90^\circ; L = 3; W = 2$ .

As to the surface deformation due to more general polygon-shaped faults, we can use the results by Comminou and Dundurs (1975). They gave the expressions of displacement and strain at the free surface of a half-space for an angular dislocation. Any polygon-shaped faults (shear or tensile) can be constructed by a superposition of a finite number of angular dislocations.

All the formulas obtained here are composed of the terms of two kinds; ones independent of the medium constants,  $\lambda$  and  $\mu$  and the others dependent on them. The latter which are denoted by  $I$ ,  $J$ , or  $K$  appear in the same fashion in the formulas of dip-slip case and tensile-fault case. This can be realized by an analogy with the  $P - SV$  coupling in the seismic wave theory, whereas the  $SH$  wave corresponds to the strike-slip case. It is clear that the deformation fields produced by a vertical fault of dip-slip type and the ones produced by a horizontal fault of any type do not depend on the medium constants,  $\lambda$  and  $\mu$ .

The  $z$  direction strain components were not given in the preceding sections, but they can be easily found as follows using the boundary conditions at the free surface.

$$\left\{ \begin{array}{l} \frac{\partial u_x}{\partial z} = - \frac{\partial U_z}{\partial x} \\ \frac{\partial u_y}{\partial z} = - \frac{\partial U_z}{\partial y} \\ \frac{\partial u_z}{\partial z} = - \frac{\lambda}{\lambda + 2\mu} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right). \end{array} \right. \quad (42)$$

To assist the development of a computer program based upon these expressions, several numerical results to check it are listed in Table 2. Here, case 1 is for the point source, and the others are for the finite rectangular sources. A medium is assumed to be  $\lambda = \mu$  in the all cases, and the results are presented in the unit of  $U\Delta\Sigma$  in case 1 and in the unit of  $U$  in the others, where  $U$  stands for  $U_1$ ,  $U_2$ , or  $U_3$ .

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