

ОПРЕДЕЛЕНИЕ ПАРАМЕТРОВ ИСТОЧНИКА ЦУНАМИ ПО
ИНСТРУМЕНТАЛЬНЫМ ЗАПИСЯМ НА ОСНОВЕ МЕТОДА R-РЕШЕНИЙ:
СВЯЗЬ ПАРАМЕТРА R И ПРОСТРАНСТВЕННОЙ ДИСКРЕТИЗАЦИИ
ИСТОЧНИКА, ВЫБОР ОПТИМАЛЬНОЙ СИСТЕМЫ НАБЛЮДЕНИЯ

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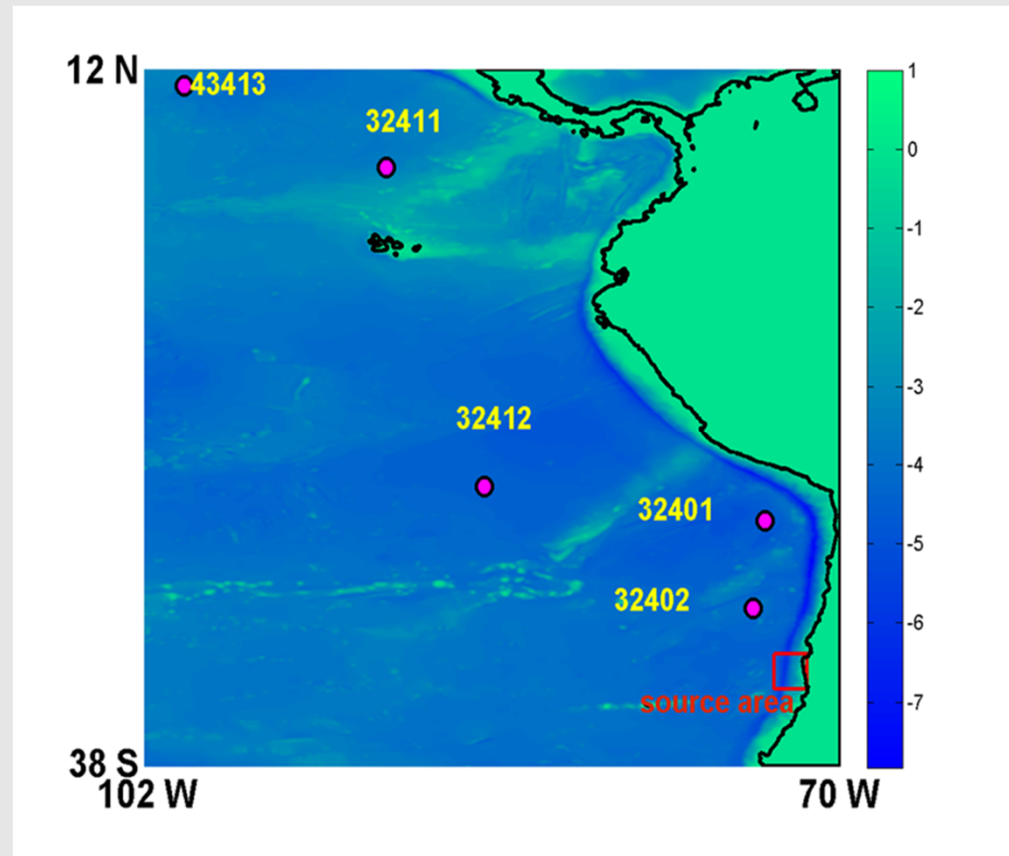
Outline

- 1. Methodology: the r-solution method**
- 2. The algorithm**
- 3. The method of determination of the optimal inversion parameters in recovering the tsunami source by the r-solution method**
- 4. Recovering the initial tsunami waveform: the case of 16 September 2015 Chile Tsunami**
- 5. Conclusion and outlook**

$$A\varphi = \bar{\eta}_0$$

The operator A is a compact linear operator

$$\eta(x, y, t)|_M = \eta_0(x_i, y_i, t), \quad M = \{(x_i, y_i), i=1, \dots, P\}$$



The calculation domain Φ and the source area Ω (red)

$$\eta_{tt} = \nabla^T (gh(x, y) \nabla \eta) \quad (1)$$

The initial conditions:

$$\eta|_{t=0} = \varphi(x, y); \quad \eta_t|_{t=0} = 0 \quad (2)$$

The boundary conditions: (3)

$$\frac{\partial \eta}{\partial \vec{n}} = 0 \quad \text{on the solid boundary}$$

$$-c\vec{V} \cdot \vec{n} - \eta_{tt} + \frac{c^2 \partial^2 \eta}{2\partial \vec{t}^2} |_{\Gamma} = 0 \quad \text{on the open boundary}$$

$$\eta(x, y, t)|_M = \eta_0(x_i, y_i, t), \quad M = \{(x_i, y_i), i=1, \dots, P\} \quad (4)$$

$\eta(x, y, t)$ the water elevation above the mean level

$\varphi(x, y) \in \Omega$ the initial sea surface displacement

$c(x, y) = \sqrt{gh(x, y)}$ the wave phase velocity

g the gravity acceleration

$h(x, y)$ the depth of the ocean

$\Omega = \{(x, y) \in [(0, l_1) \times (0, l_2)]\}$ the tsunami source area
 $\Omega \subset \Phi$

$\eta_0(x_i, y_i, t)$ the observed tsunami waveform

1. Methodology / the r-solution method

➔ singular system of compact operator $A : (\bar{v}_i, \bar{u}_i, s_i)$
 \bar{v}_i, \bar{u}_i are the left and the right singular vectors,
 $\{s_i\}$ are its singular values, $i \rightarrow \infty, s_i \rightarrow 0$

➔ $A\bar{v}_i = s_i\bar{u}_i; \quad i. e. A = U\Sigma V^T; \quad \Sigma = \begin{pmatrix} s_1 & \dots & \vdots \\ \vdots & s_2 & \vdots \\ \dots & \dots & \ddots \end{pmatrix}$

$$\varphi(x, y) = \sum_{i=1}^{\infty} \frac{(\bar{\eta}_0 + \bar{\varepsilon}, \bar{u}_i)}{s_i} \bar{v}_i, \quad s_i / (\bar{\varepsilon}, \bar{u}_i) \rightarrow 0, \quad i \rightarrow \infty, \quad \bar{\varepsilon} - \text{data error}$$

r-solution

$$\varphi^{[r]}(x, y) = V\bar{\Sigma}_r^{-1}P_r(U^T\bar{\eta}_0)$$

➔

$$\bar{\Sigma}_r = \begin{pmatrix} s_1 & & \dots & 0 \\ & s_2 & & \\ \vdots & & \ddots & \vdots \\ 0 & & \dots & s_r \end{pmatrix}$$

$$\varphi^{[r]}(x, y) = \sum_{i=1}^r \frac{(\bar{\eta}_0 + \bar{\varepsilon}, \bar{u}_i)}{s_i} \bar{v}_i$$

2. The algorithm/Discrete inverse problem

$$\varphi(x, y) = \sum_{m=1}^M \sum_{n=1}^N c_{mn} \sin \frac{\pi m}{l_1} x \cdot \sin \frac{\pi n}{l_2} y = \sum_{m=1}^M \sum_{n=1}^N c_{mn} \varphi_{mn}(x, y)$$

$$\Omega = [0, l_1] \times [0, l_2] \quad \bar{c} = \{c_{mn}\}_{m=1, n=1}^{m=M, n=N} \quad ; \quad \bar{\eta}_0 = \{\eta_0(x_p, y_p, t_j)\}, \quad p=1, \dots, P; j=1, \dots, N_t$$

$$\mathbf{A}\bar{c} = \bar{\eta}_0$$

The matrix \mathbf{A} : $(P \times N_t) \times (M \times N)$; $\eta|_{t=0} = \varphi_{mn}$;

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

r - solution
$$\bar{c}^{[r]} = \sum_{j=1}^r \frac{(\bar{\eta}_0 + \bar{\varepsilon}) \cdot \bar{u}_j}{s_j} \bar{v}_j,$$

$$r = \max \left\{ k: \frac{s_k}{s_1} \geq d \right\}$$

2. The numerical simulation includes the following steps:

1) First, the matrix **A** is numerically computed by solving the forward problem with every spatial harmonic

$\{\varphi_{mn}\}, m = 1, \dots, M; n = 1, \dots, N$ as a source.

2) Further, the standard **SVD** procedure is applied. The singular system $A : (\bar{v}_i, \bar{u}_i, s_i)$ is obtained

3) Setting the parameter **d** one can determine a possible value

$$r = \max \left\{ k : \frac{s_k}{s_1} \geq d \right\}$$

4) Computing the coefficients $\{c_{mn}\}$

5) Computing the **r-solution**

$$\varphi^{[r]}(x, y)$$

6) Computing the marigrams at the points corresponding to the recorders of the observation system and at the **predetermined points of the calculation domain**

Voronina T.A. ,(2004), Determination of the spatial distribution of the oscillation sources by the remote measurements on a finite set of points, Sib. J. Calc. Math.,7(3),pp. 203-211 (rus).

Cheverda V.A. and Kostin V.I., (2010), r-pseudoinverse for compact operator, SEMR,v.7, p258=282.

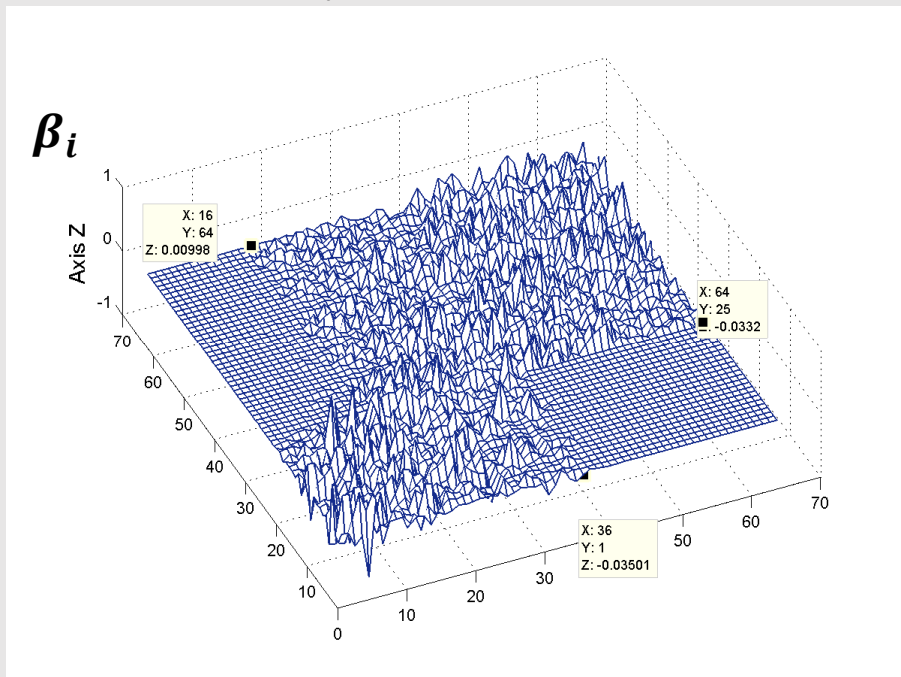
Voronina, T. A., Tcheverda, V.A., and Voronin, V. V. ,(2014), Some properties of the inverse operator for a tsunami source recovery, Siberian Electronic Mathematical Reports, vol. 11, pp. 532-547.

Voronin,V.V., Voronina,T.A., and Tcheverda,V.A.,(2015), Inversion method for initial tsunami waveform reconstruction, Nat. Hazards Earth Syst. Sci., vol. 15, pp. 1251-1263.

Voronina, T.A., Romanenko, A.A., (2016), The New Method of Tsunami Source Reconstruction With r-Solution Inversion Method. Pure and Applied Geophysics, 173(12), 4089 - 4099.

Voronina, T. A., Voronin, V. V. , Tcheverda, V.A., (2019) The 2015 Illapel Tsunami Source Recovery by Inversion of DART Tsunami Waveforms Using the R-Solution Method//Pure and Applied Geophysics, Springer Nature Switzerland AG, <https://doi.org/10.1007/s00024-019-02100-y>

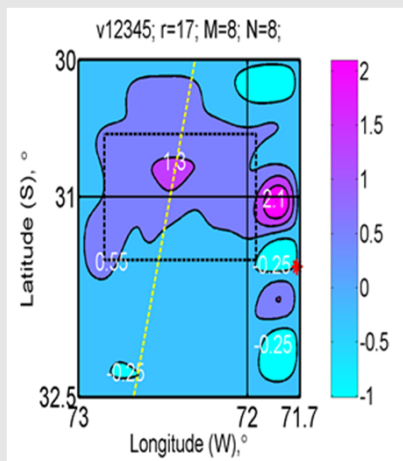
a)



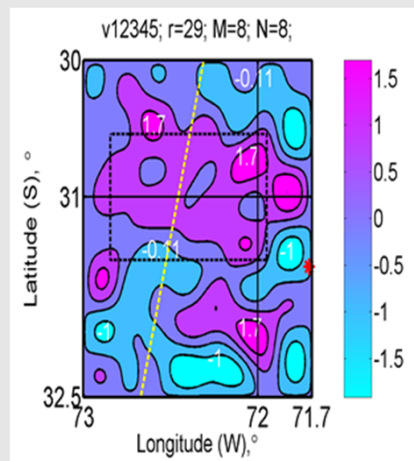
a) The right singular vectors in terms of the spatial harmonics:

$$\bar{v}_i = \sum_k^{N \times M} \beta_{ik} \varphi_k; \quad M \times N = 64;$$

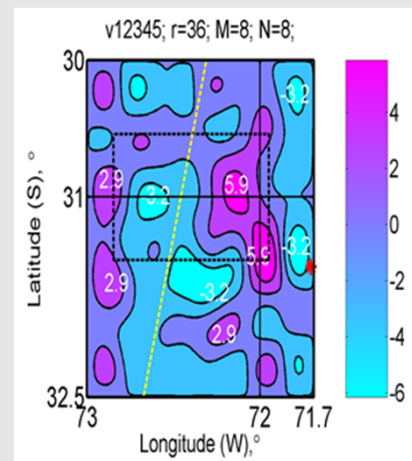
$$\varphi_k = \varphi_{mn}; \quad k = n + (m - 1) \times N; \quad m = 1, \dots, M; \quad n = 1, \dots, N;$$



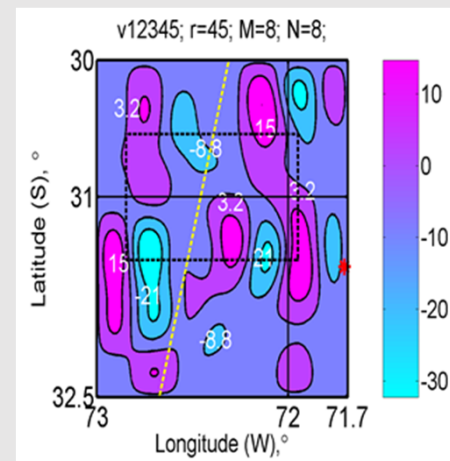
$\varphi_{max}=2.91;$
 $\varphi_{min}= - 1.04;$



$\varphi_{max}=2.64;$
 $\varphi_{min}= -2.16;$



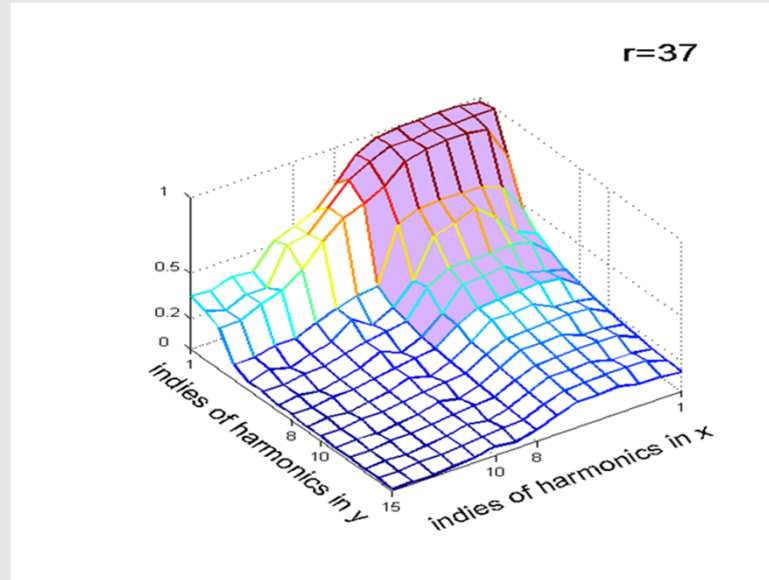
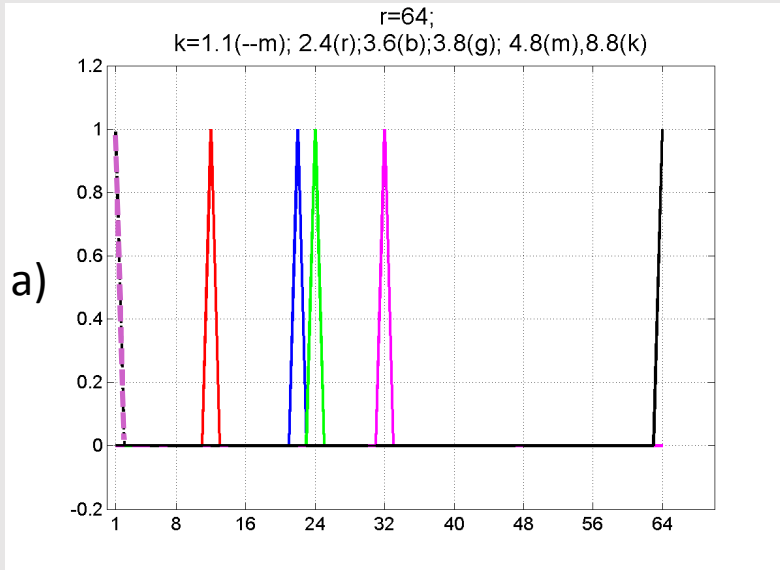
$\varphi_{max}=9.01;$
 $\varphi_{min}= -6.25;$



$\varphi_{max}=27.01;$
 $\varphi_{min}= -33.64;$

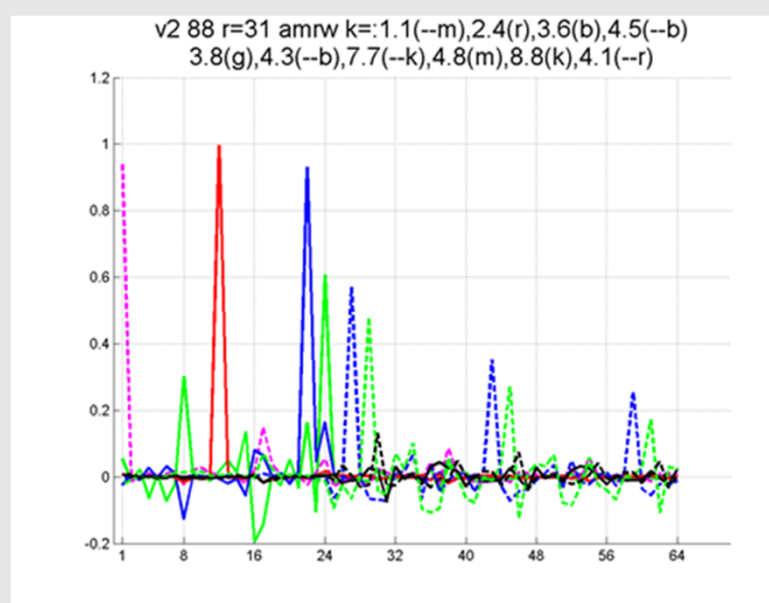
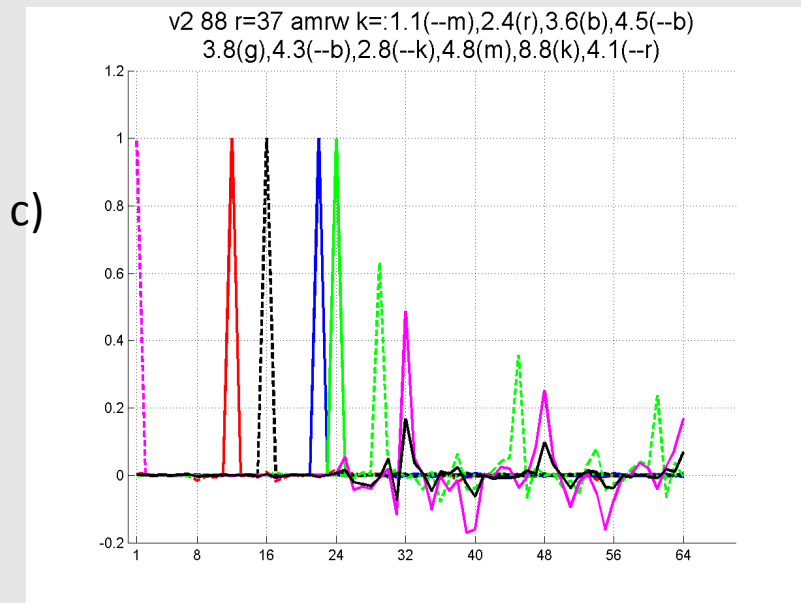
$$\varphi(x, y) = \sum_{m=1}^M \sum_{n=1}^N c_{mn} \sin \frac{\pi m}{l_1} x \cdot \sin \frac{\pi n}{l_2} y = \sum_{m=1}^M \sum_{n=1}^N c_{mn} \varphi_{mn}(x, y)$$

$$\varphi_k = \varphi_{mn}; \quad k = n + (m-1) \times N; m=1, \dots, M; n=1, \dots, N;$$



The cosine of the angle between each harmonic and a subspace of r-solutions, r=37

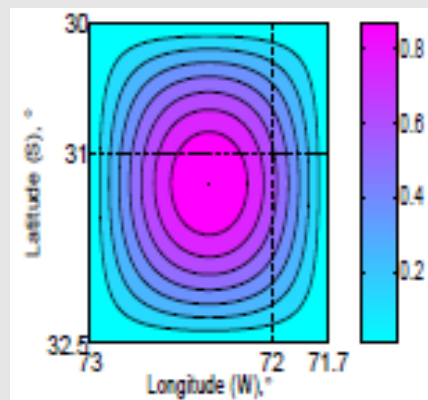
- K=1 -- {1.1}
- K=12 {2.4}
- K=16 --
- 4.3} K=22
- {3.6} K=24
- {3.8} K=25
- {4.1}
- ; K=27 --
- (4.3}
- K=32 {4.8}
- K=64 {8.8}



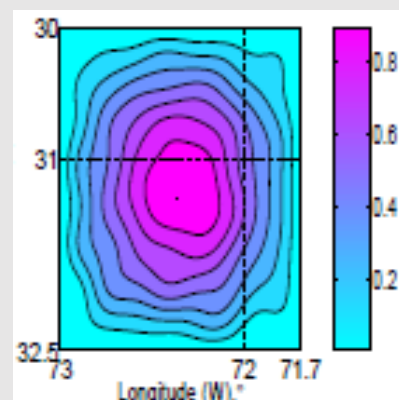
2. Регуляризация оператора в данном случае осуществляется путем сужения оператора на подпространство, являющееся линейной оболочкой его r первых правых сингулярных векторов. Тем самым подпространство, в котором ищется решение, выбирается на основе анализа свойств сингулярного спектра оператора, которые определяются системой наблюдения и батиметрией.

Неизвестная функция $\varphi(x, y)$ ищется в виде конечного отрезка ряда Фурье по пространственным гармоникам с неизвестными коэффициентами. От количества гармоник зависит как время вычисления матрицы оператора прямой задачи, которая представляет собой отклики на каждую гармонику в каждой точке системы наблюдения, так и результат инверсии в целом.

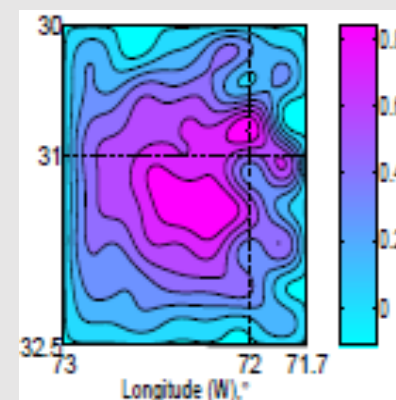
В работе исследовались структура правых сингулярных векторов полученной матрицы и влияния усечения SVD на восстановление гармоник в зависимости от их номеров. Установлено, что при проектировании высокочастотные гармоники плохо восстанавливаются, появляются дополнительные осцилляции, а при дальнейшем уменьшении r - эти явления усиливаются и, как следствие появляются артефакты в решении.



$r=64$



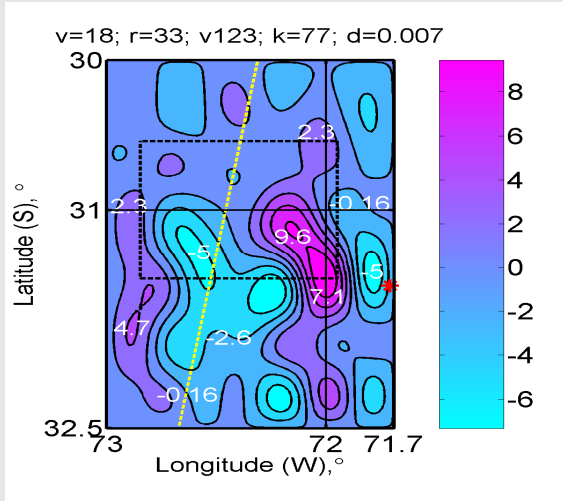
$r=37$



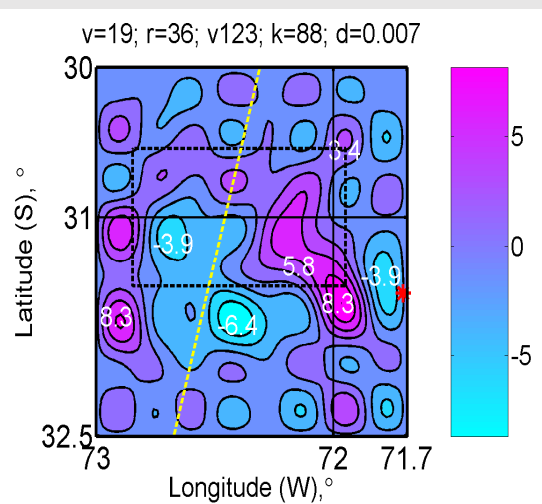
$r=31$

Восстановление
1-ой гармоник
при разных r

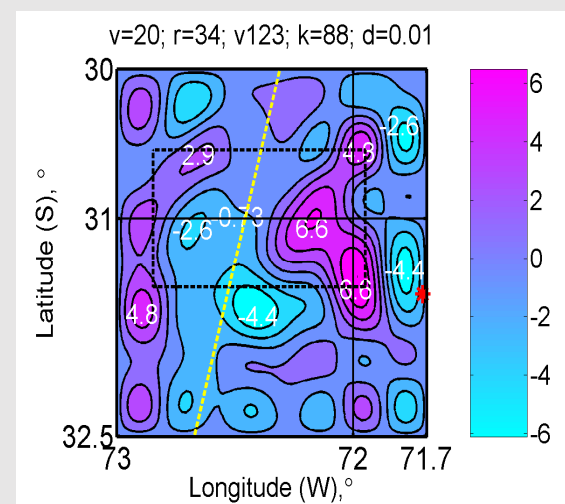
Расчеты прямой задачи проводились на FORTRAN коде на HP DL980 G7 сервере с использованием 2-х по 10 ядер Intel Xeon E7-4870 процессоров - 20 ядер в целом.



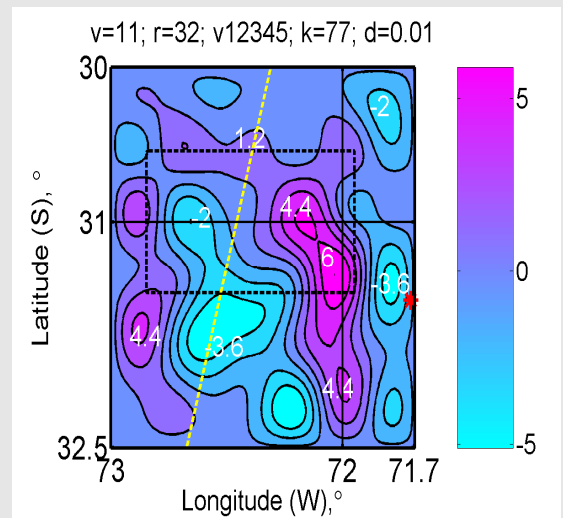
$$\varphi_{max} = 12.01; \varphi_{min} = -7.46$$



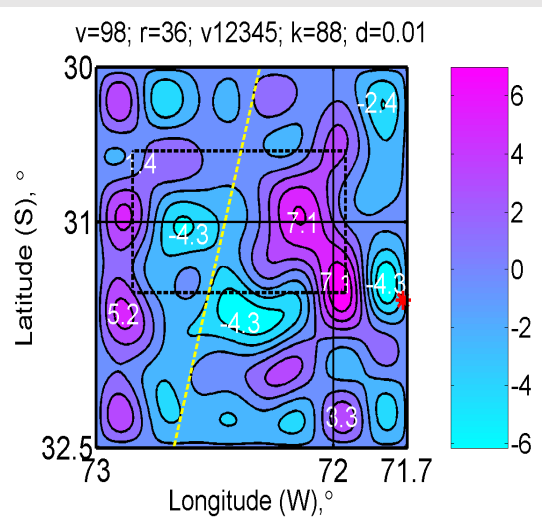
$$\varphi_{max} = 10.73; \varphi_{min} = -8.8;$$



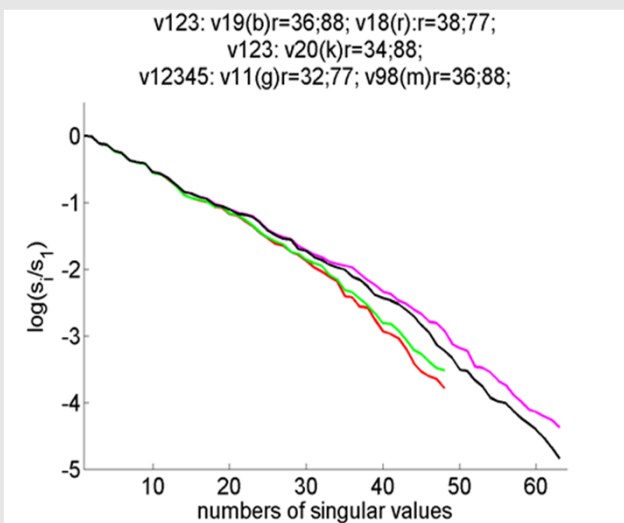
$$\varphi_{max} = 7.46; \varphi_{min} = -6.2;$$

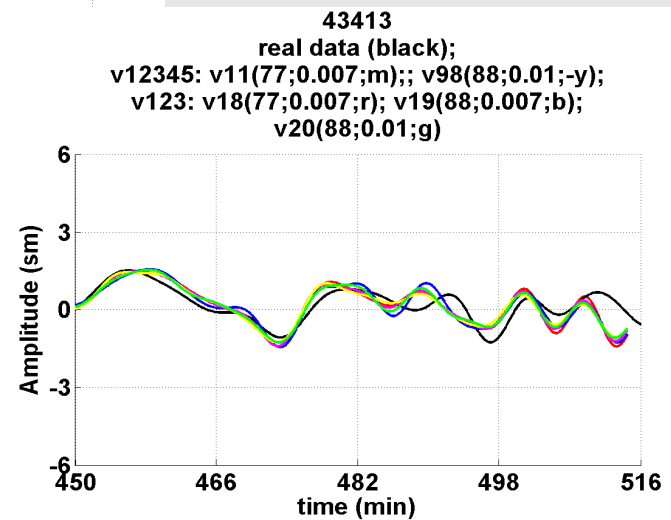
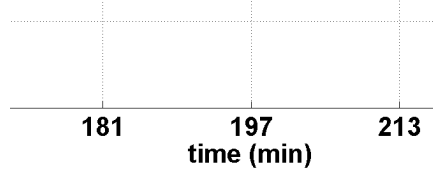
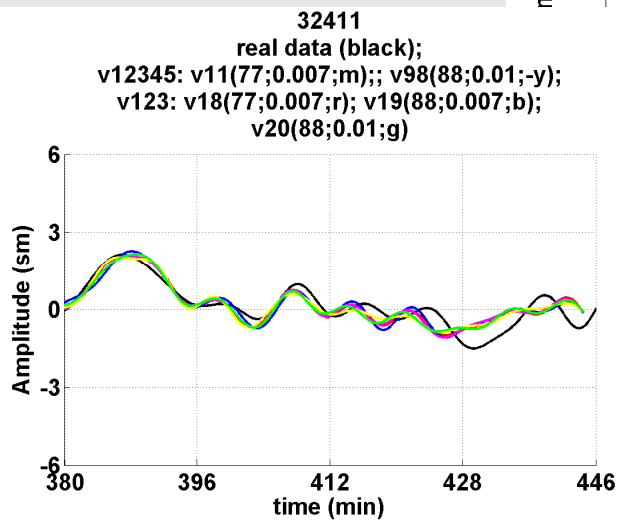
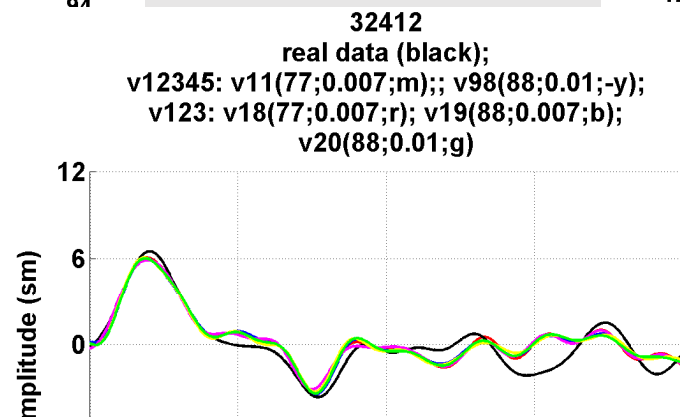
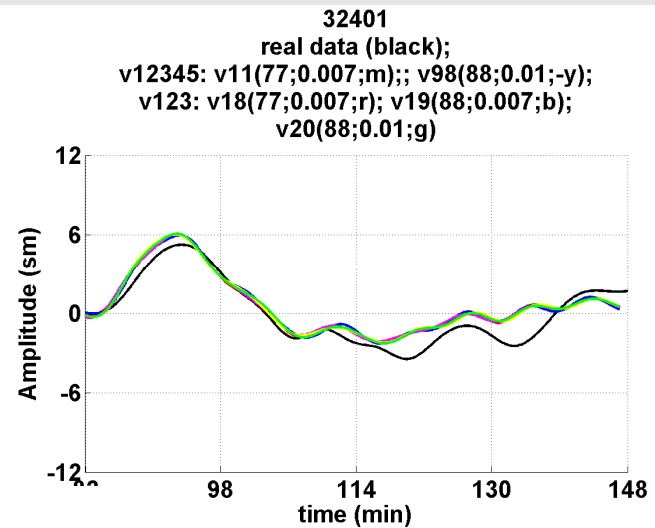
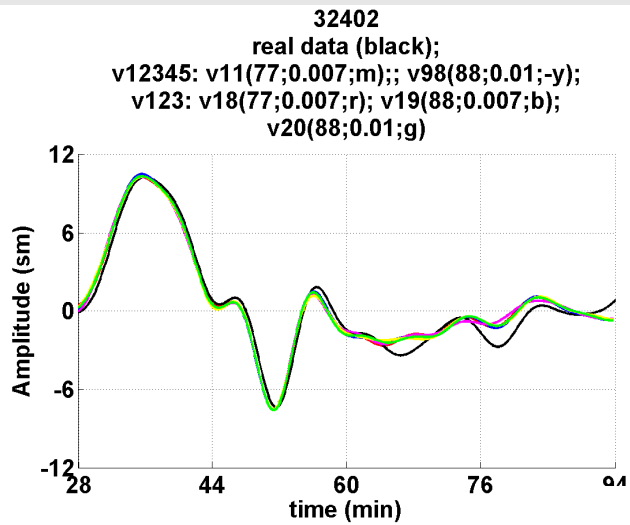


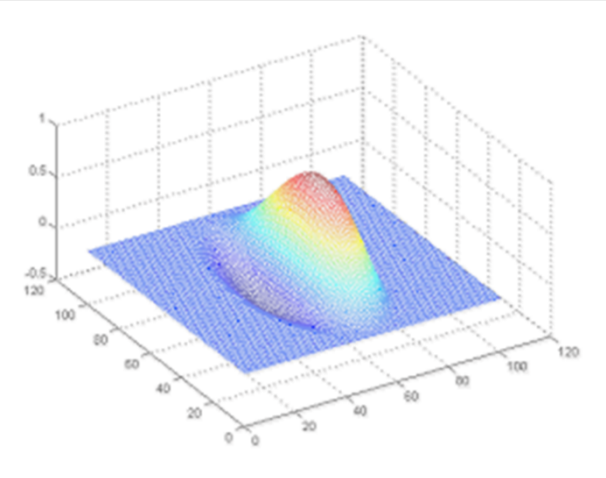
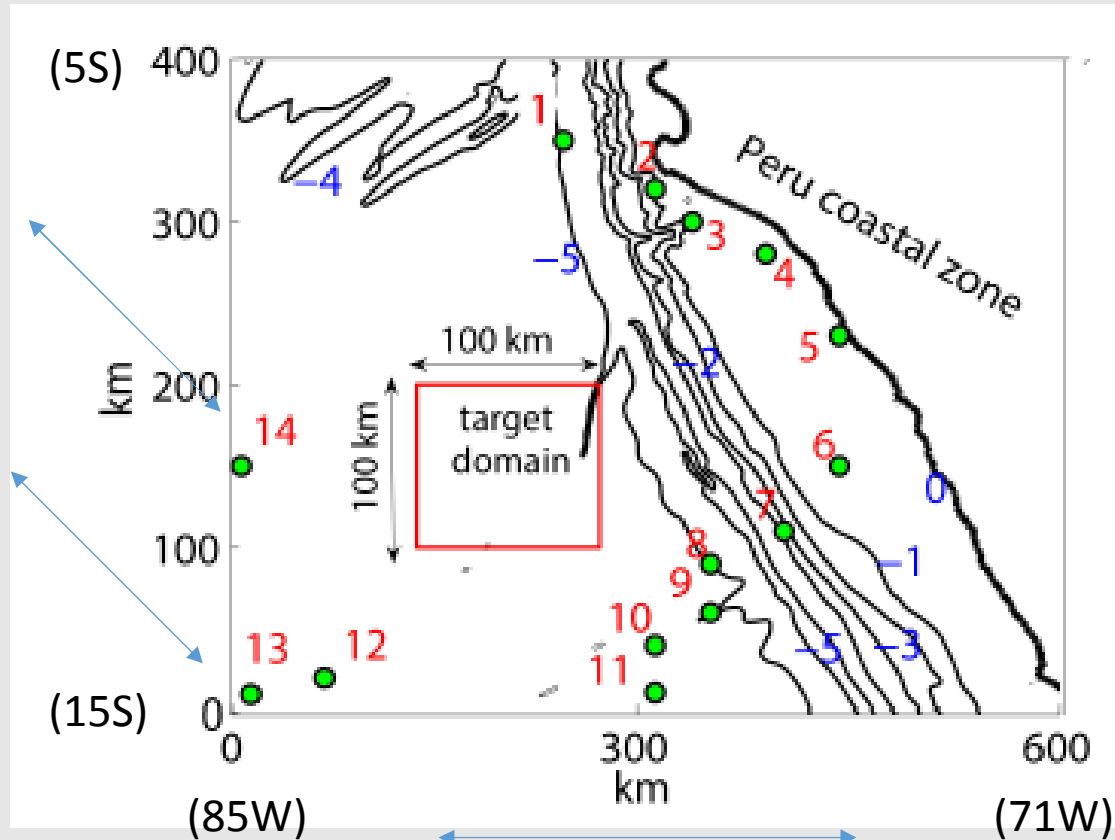
$$\varphi_{max} = 7.585; \varphi_{min} = -5.195;$$



$$\varphi_{max} = 9.01; \varphi_{min} = -6.25;$$



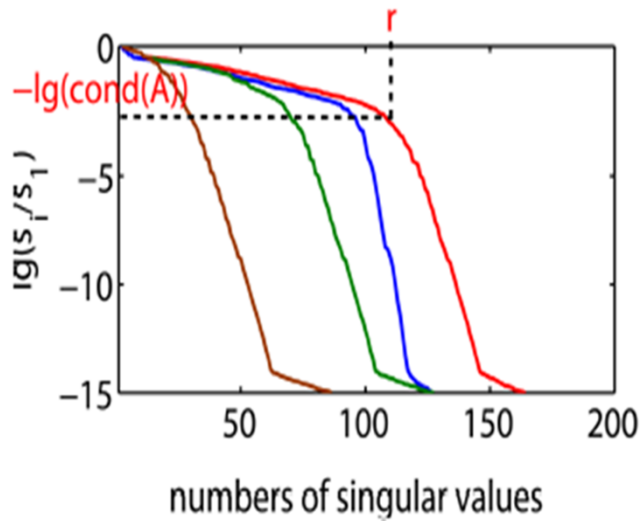




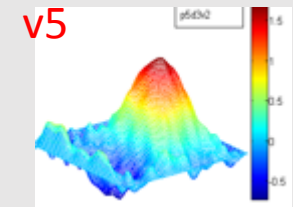
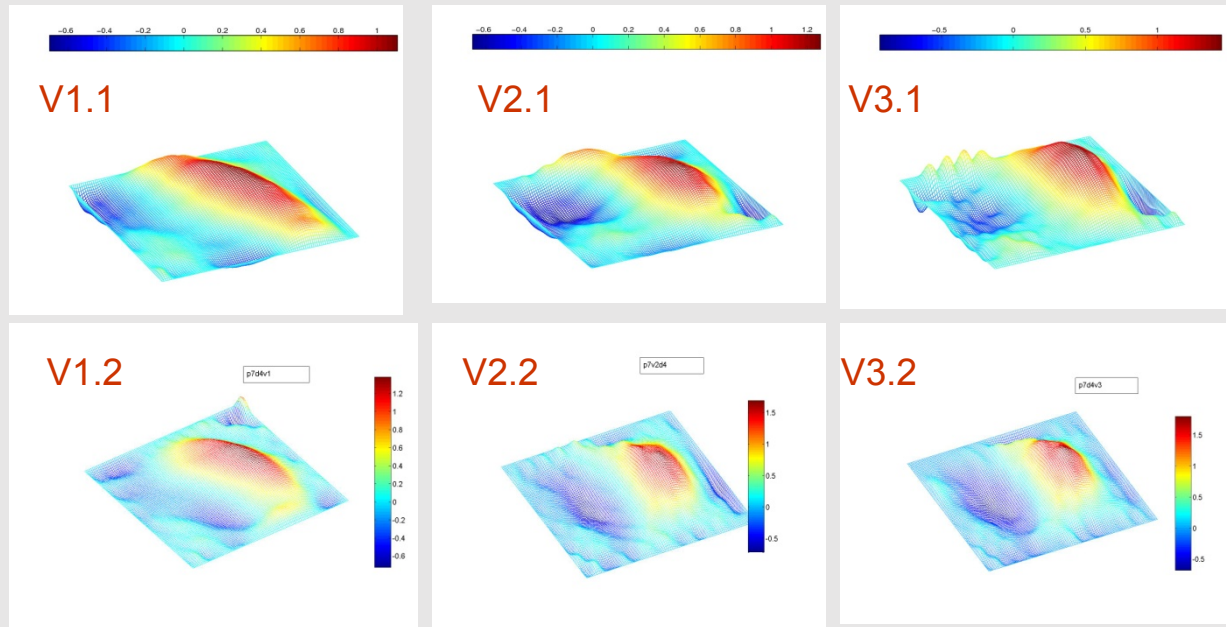
$$\varphi_{max}=1.959;$$

$$\varphi_{min}=-0.67$$

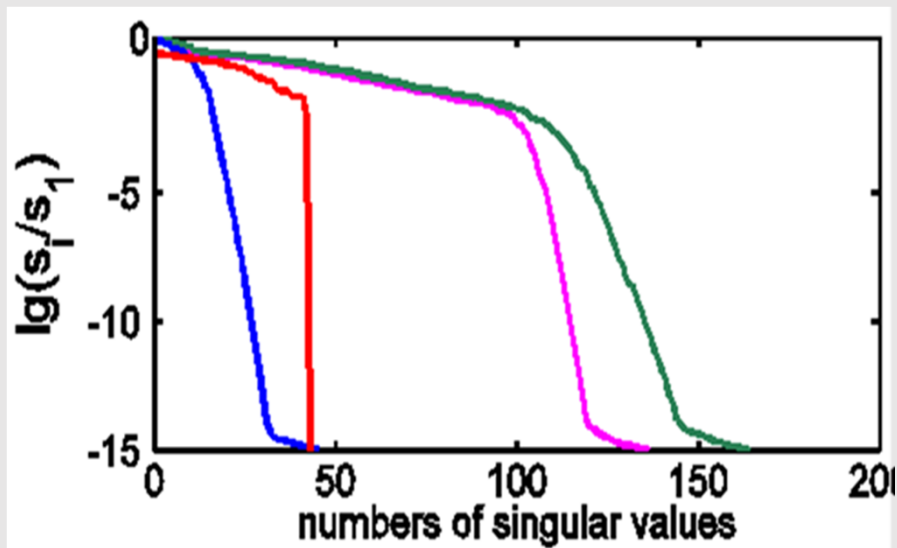
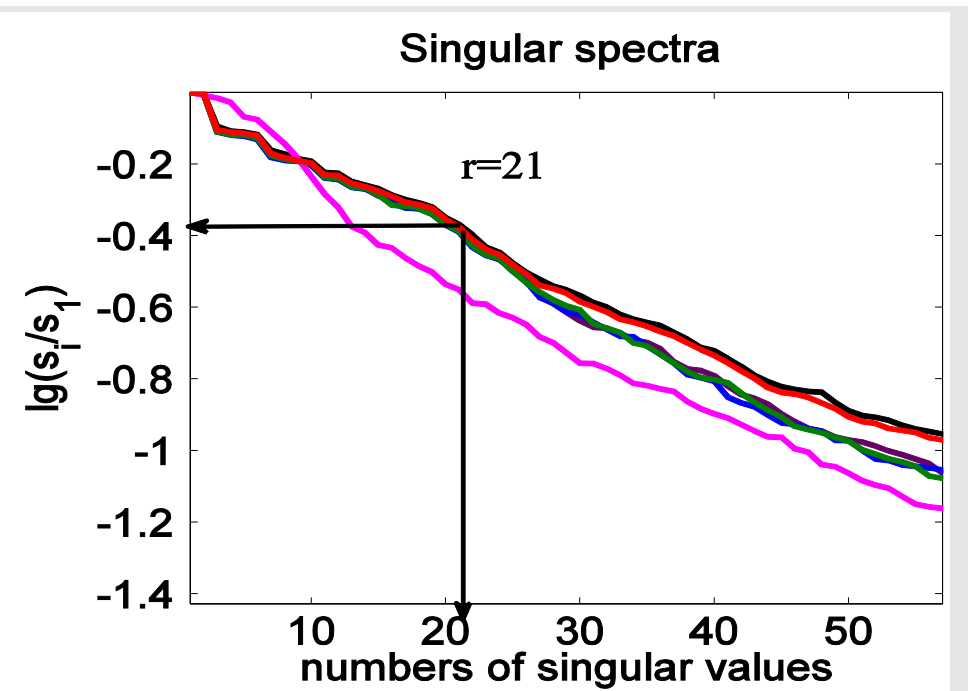
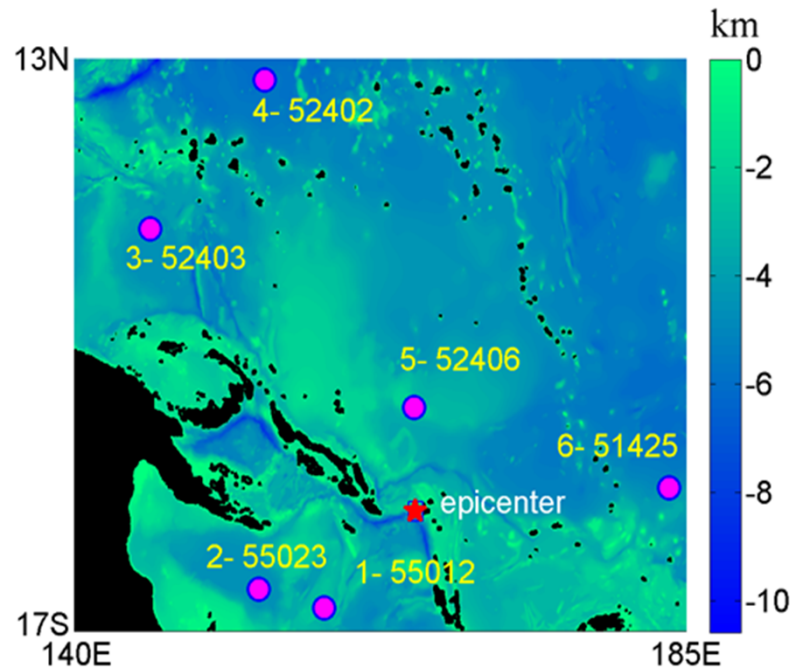
The location of receivers on direct and reflected rays corresponding to the direction of the strongest variability of the dipole source have the greatest effect for the inversion result. As is shown in the numerical experiments, it is this set of receivers that provides the best results of the inversion process.



V1 (brown), V2 (red); V3 (blue), V4 (green)



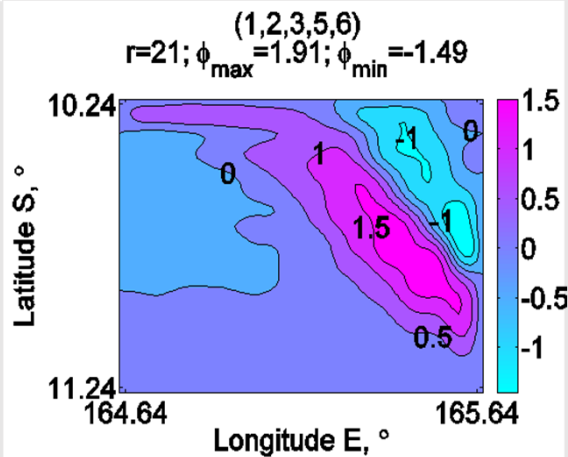
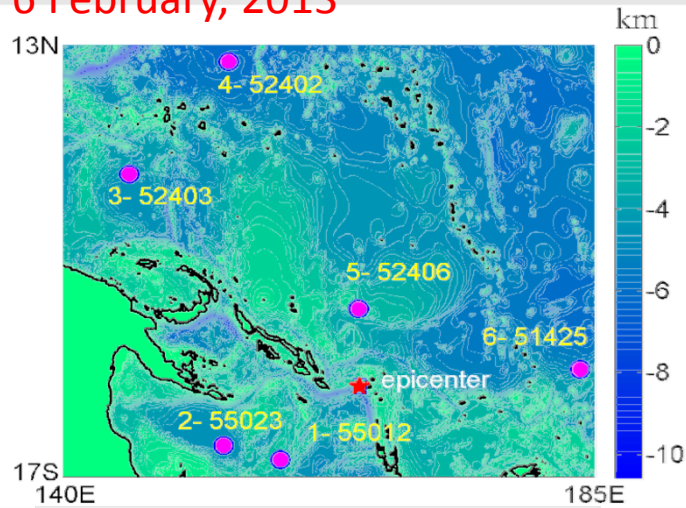
M	Cond	R	err%	used receivers (red)
v5	1000	57	41	1,2,3,4,5,6,7,8,9,10,11,12,13,14
V1.1	100	23	62.8	1,2,3,4,5,6,7,8,9,10,11,12,13,14
V1.2	10000	32	52	1,2,3,4,5,6,7,8,9,10,11,12,13,14
v2.1	100	73	37.7	1,2,3,4,5,6,7,8,9,10,11,12,13,14
V2.2	1000	92	32.6	1,2,3,4,5,6,7,8,9,10,11,12,13,14
V3.1	100	65	45.9	1,2,3,4,5,6,7,8,9,10,11,12,13,14
V3.2	10000	99	26.7	1,2,3,4,5,6,7,8,9,10,11,12,13,14
v4	1000	104	60	1,2,3,4,5,6,7,8,9,10,11,12,13,14



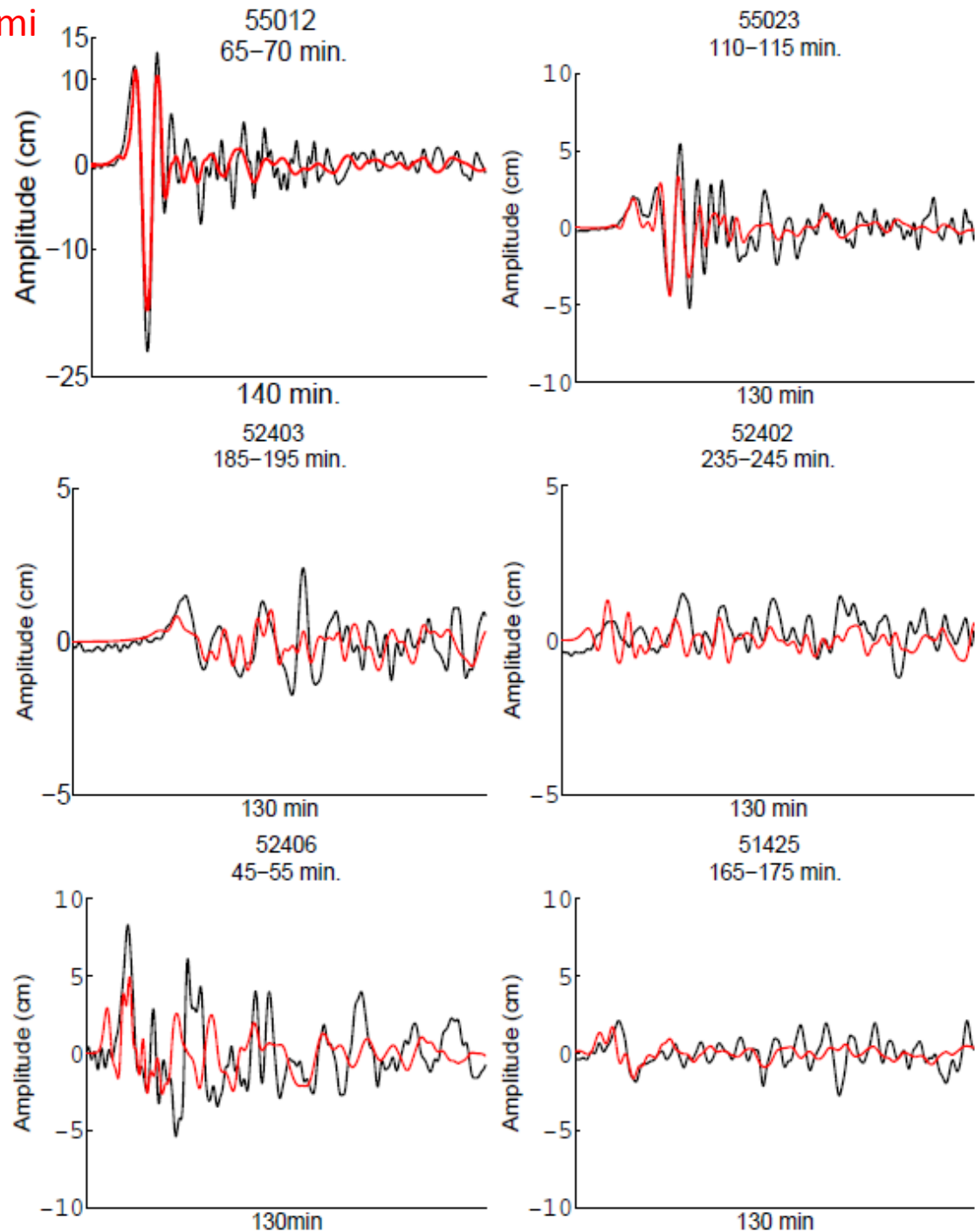
Graphs of singular values in the common logarithm scale of the matrix A obtained in the inversion with the different sets of the receivers :

- {4, 5, 6} (magenta),
- {1, 2, 5} (dark magenta),
- {1, 3, 5} (blue), {1, 5, 6} (green),
- {1, 2, 3, 5, 6} (red) and
- {1, 2, 3, 4, 5, 6} (dark blue).

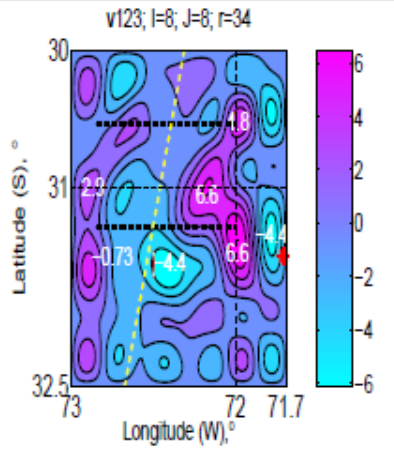
4. Application to Solomon Islands Tsunami 6 February, 2013



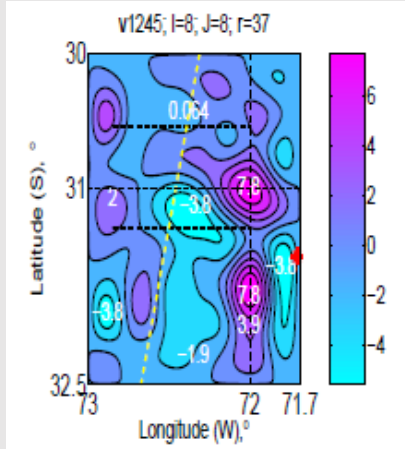
Comparison of observed marigrams (the black line) and calculated ones with five receivers numbered {1, 2, 3, 5, 6} (the red line) of the 2013 Solomon Islands tsunami.



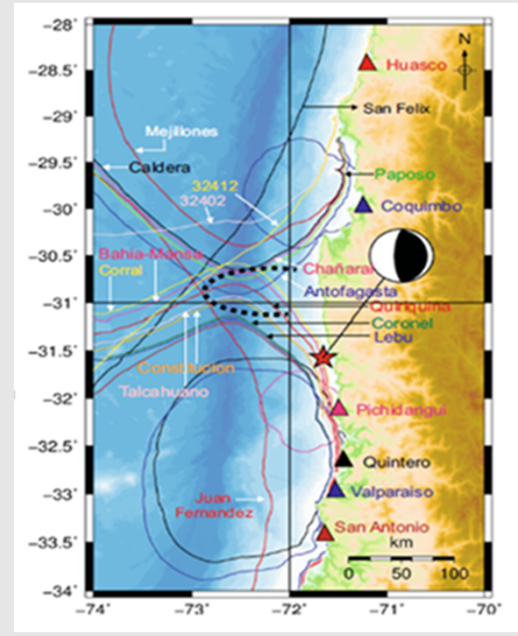
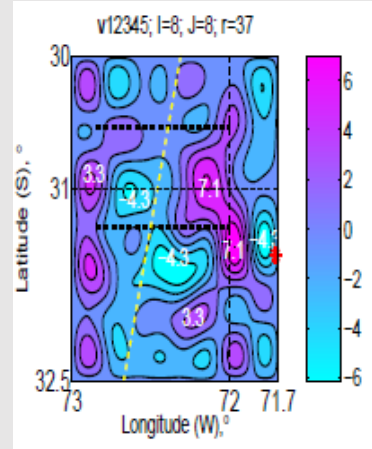
v123



v1245

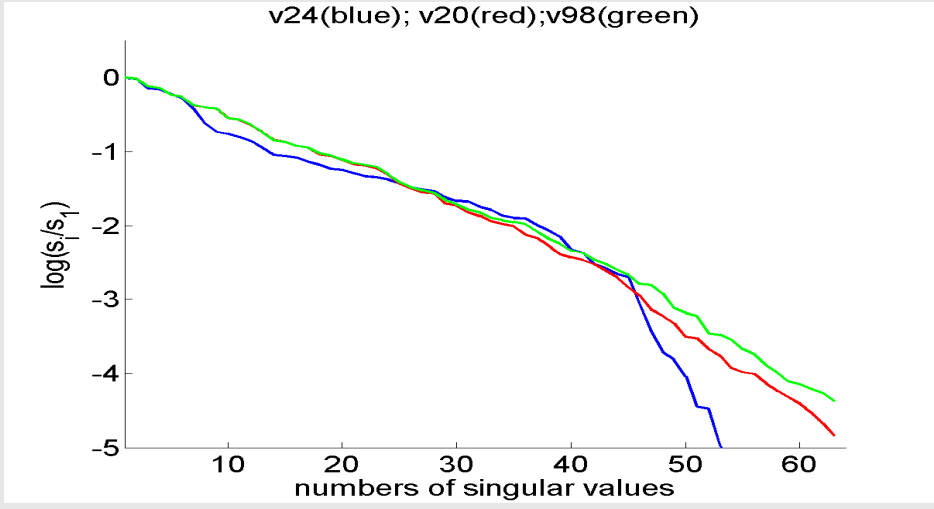


v12345



M = 8; N = 8; condA^[r] = 100;	v1245 r=37	Φ_{max} 9.67	Φ_{min} -5.7
	v12345 r=37	9.01	-6.3
	v123 r=34	7.4	-6.2

Heidarzadeh, M., Murotani, S., Satake, K., Ishibe, S., and Gusman, A. R. (2016). Source model of the 16 September 2015 Illapel, Chile, Mw 8.4 earthquake based on teleseismic and tsunami data. *Geophysical Research Letters*, 43, 643 - 650.



Graphs of the decimal logarithms of the singular values of the matrix A relative to their numbers, the calculation options: v1245 (blue); v12345 (green); v123 (red).

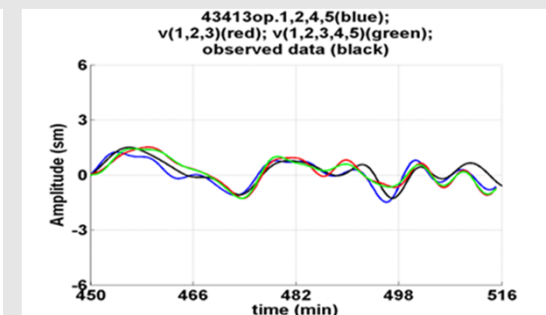
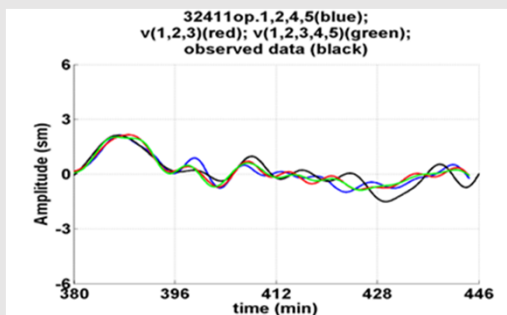
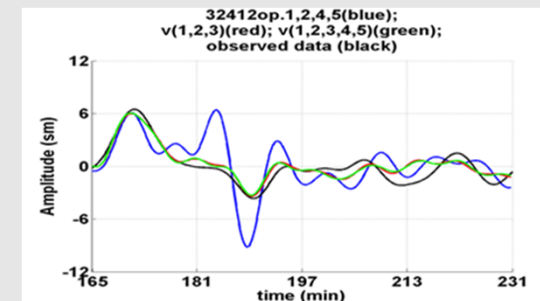
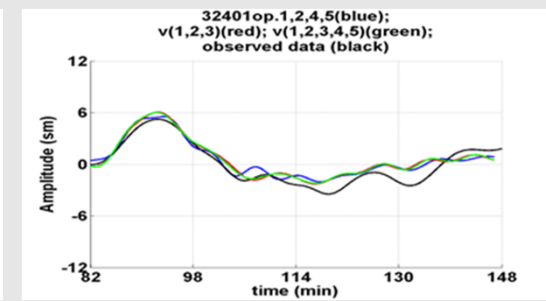
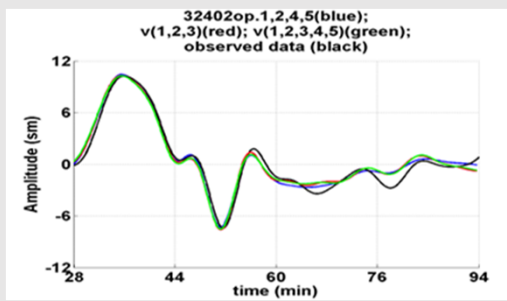
Сравнение наблюдаемых и восстановленных мареограмм.

Следует особо подчеркнуть хорошее совпадение не только мареограмм, участвовавших в инверсии, но и мареограмм в точках буев ДАРТ которые не использовались для решения обратной задачи:

- 1) в инверсии участвовали данные **1-ого, 2-ого и 3-его (цвет зеленый)** регистраторов, восстановились все мареограммы, в том числе **в 4-ом и 5-ом;**
- 2) использовались данные всех **пяти** регистраторов – **цвет красный;**
- 3)) использовались данные **четырёх** регистраторов **(цвет синий)** без данных 3-его регистратора.

Восстановление 3-ей мареограммы не происходит. Особая роль 3-его ДАРТа объясняется его расположением на направлении наибольшей изменчивости волны, что связано с конкретной батиметрией области источника.

Сравнение графиков позволяет говорить об адекватности предложенного подхода к решению обратной задачи восстановления источника цунами.



Установлено, что результат восстановления существенно зависит от пространственного распределения регистрирующей системы относительно предполагаемого очага цунами, количества регистраторов, особенностей рельефа дна и уровня шумов, а также правильно подобранного количества пространственных гармоник, используемых в представлении искомой функции.

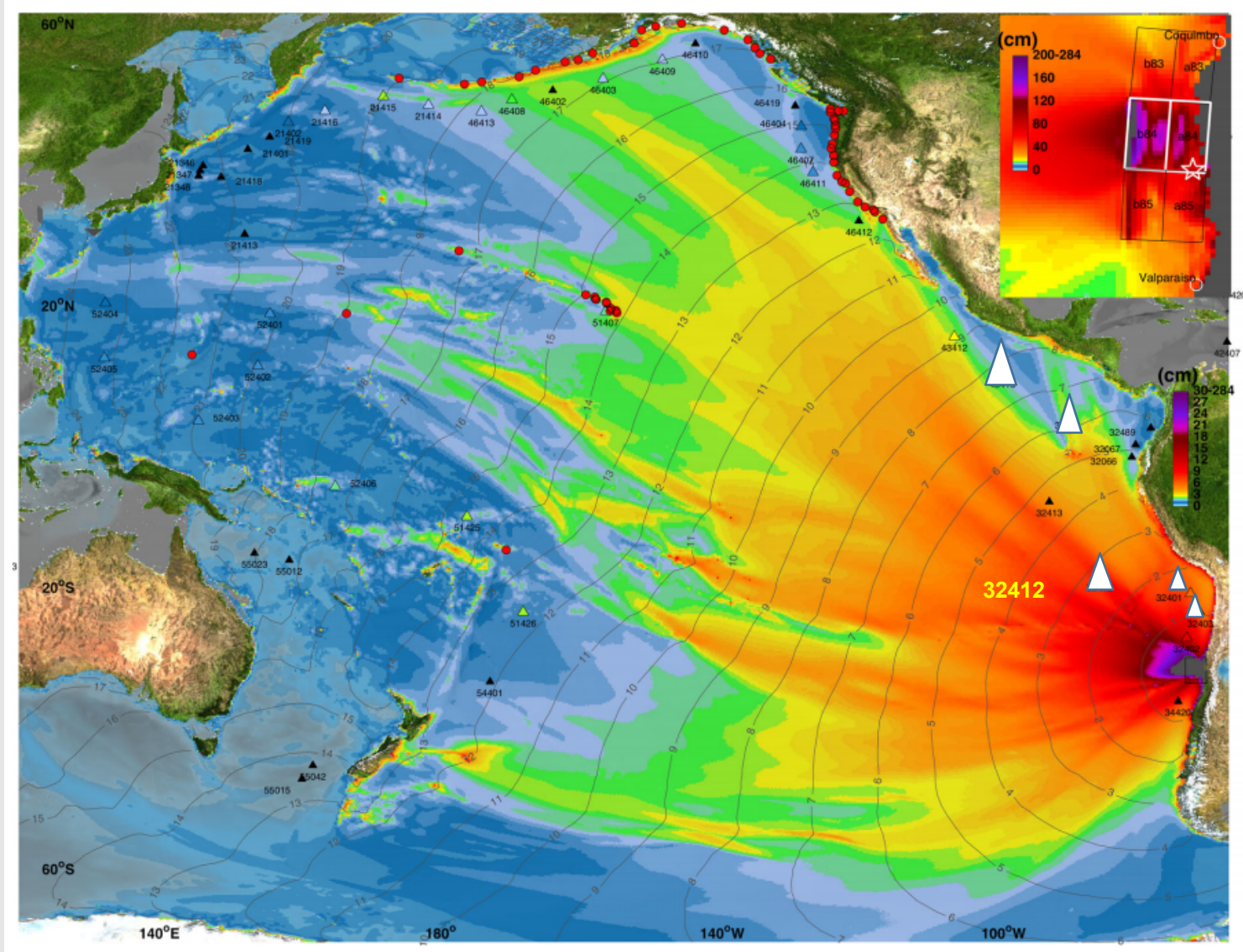


Figure 5
Energy propagation patterns guided by seafloor topography during the 2015 Chile tsunami.

Tang L., Titov V.V., Moore C., and Wei Y. (2016) Real-Time Assessment of the 16 September 2015 Chile Tsunami and Implications for Near-Field Forecast. Pure and Applied Geophysics, 173, 369 - 367.

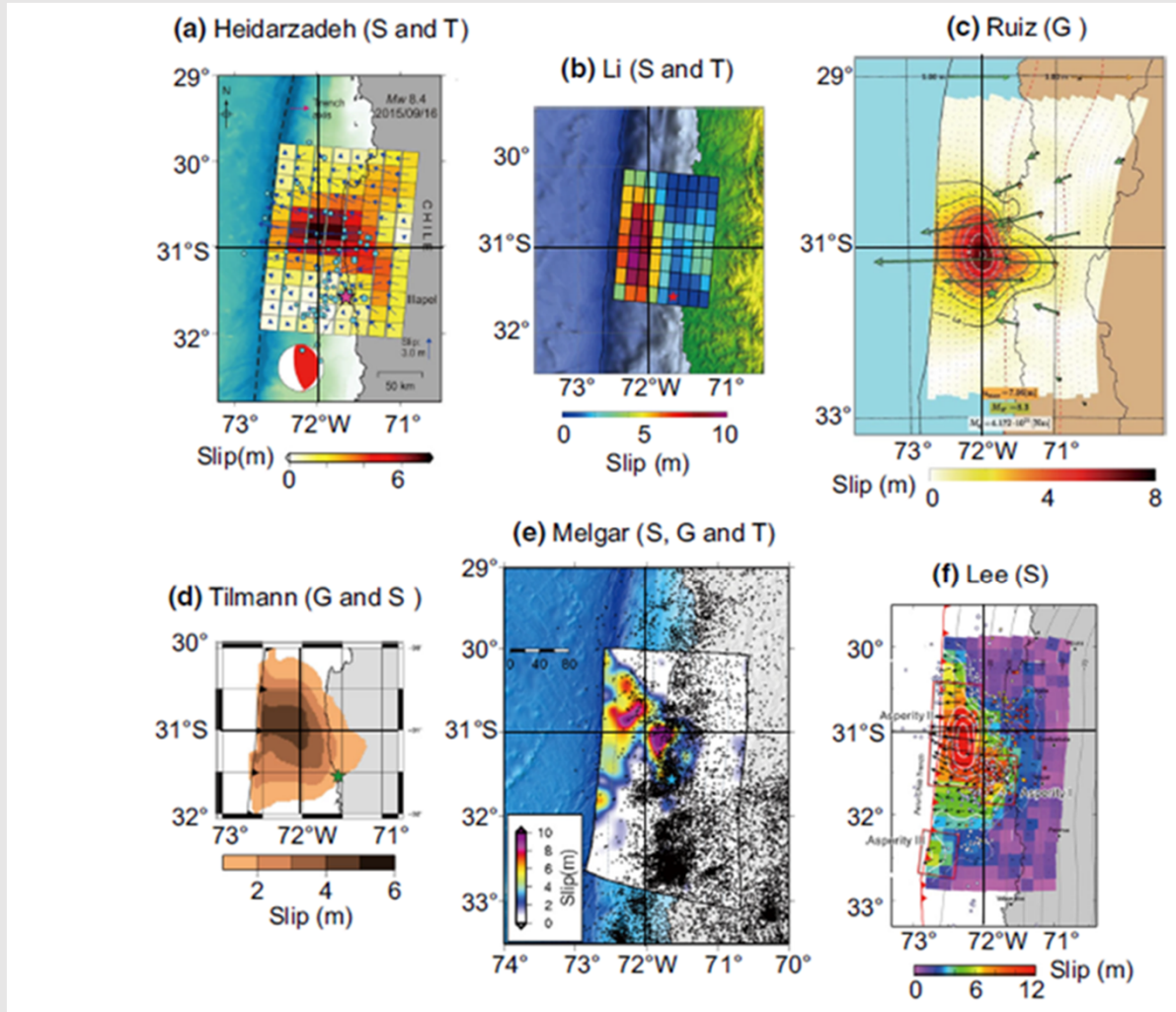
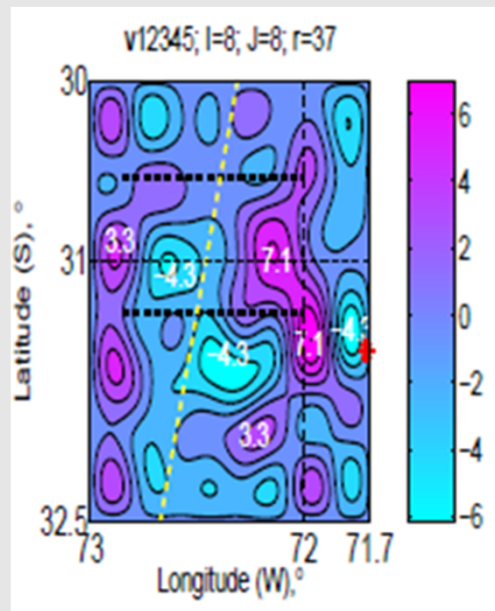
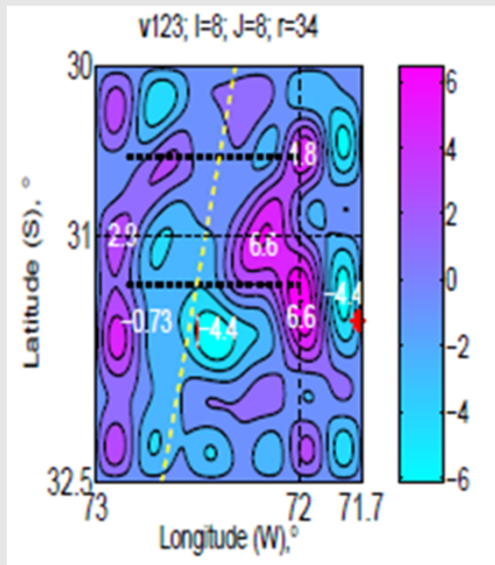


Figure 4
Six slip distribution models estimated by various data sets. S, T, and G stand for seismic, tsunami, and geodetic data, respectively. References are a Heidarzadeh et al. (2016), b Li et al. (2016), c Ruiz et al. (2016), d Tilmann et al. (2016), e Melgar et al. (2016), and f Lee et al. (2016)

Satake K., Heidarzaden M. A review of source models of the 2015 Illapel, Chile earthquake and insights from tsunami data // Pure Appl. Geophys. 2017. Vol. 174, Iss. 1. P. 1-9. doi:10.1007/s00024-016-1450-5.

5. Conclusion

1. The method proposed suppresses the instability due to the ill-posedness of the problem. The characteristic features of the r -solution obtained are largely determined by the underlying bathymetry and the observation system.
2. Analysis of the singular spectra of the matrix A makes possible to improve the quality of the inversion by selecting the most informative set of the available observation stations.
3. Based on the numerical experiments, we conclude that the method presented provides a good matching in the observed and the calculated tsunami waveforms.
4. Moreover, the algorithm provides a possibility to simultaneously restore the tsunami source and the calculated tsunami waveforms even at the points, where there are no observational data (There is no need to solve the wave propagation problem once more).
5. The study of the singular vectors allows one to obtain the desired balance with respect to the number of harmonics used.



Thank for your attention