

# Внутренние волны цунами, генерируемые при извержении подводного вулкана

**Efim Pelinovsky, T. Talipova, E. Didenkulova**

**Institute of Applied Physics, Nizhny Novgorod, Russia**



**ВЫСШАЯ ШКОЛА ЭКОНОМИКИ**  
НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ



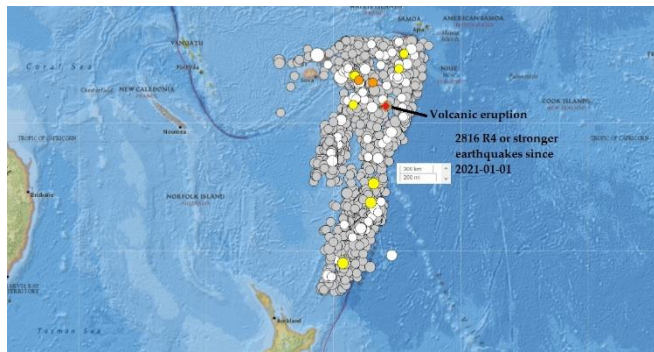
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*РНФ 24-27-00110*

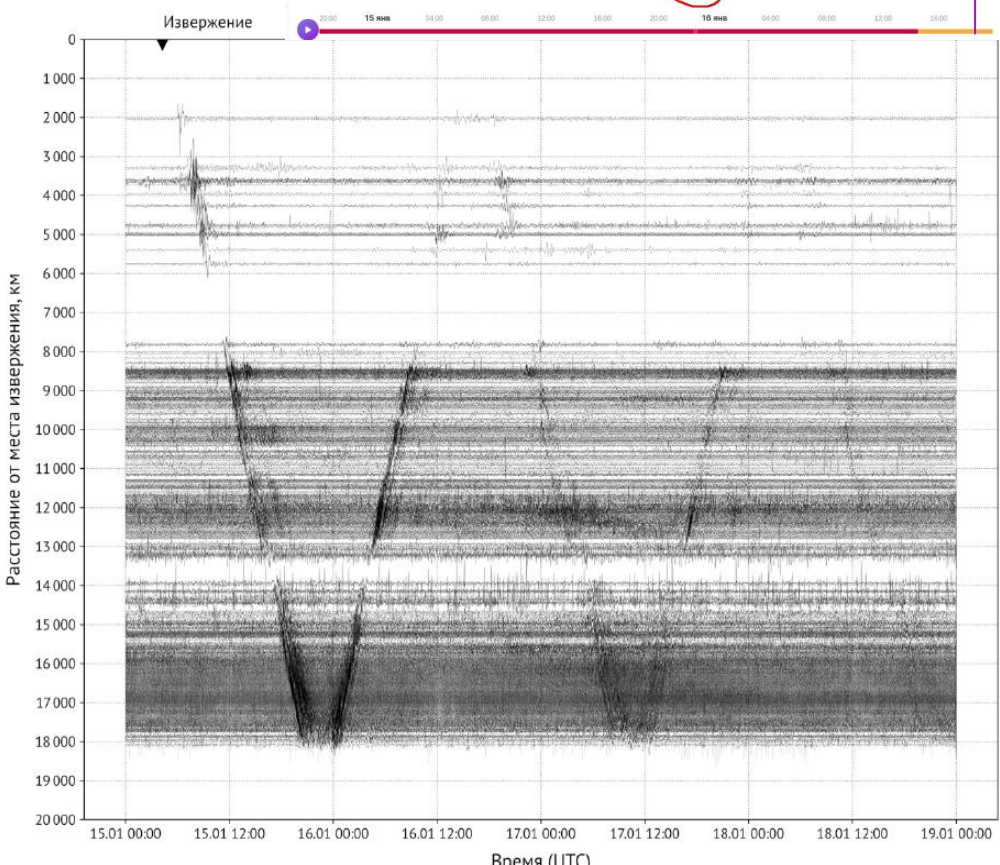
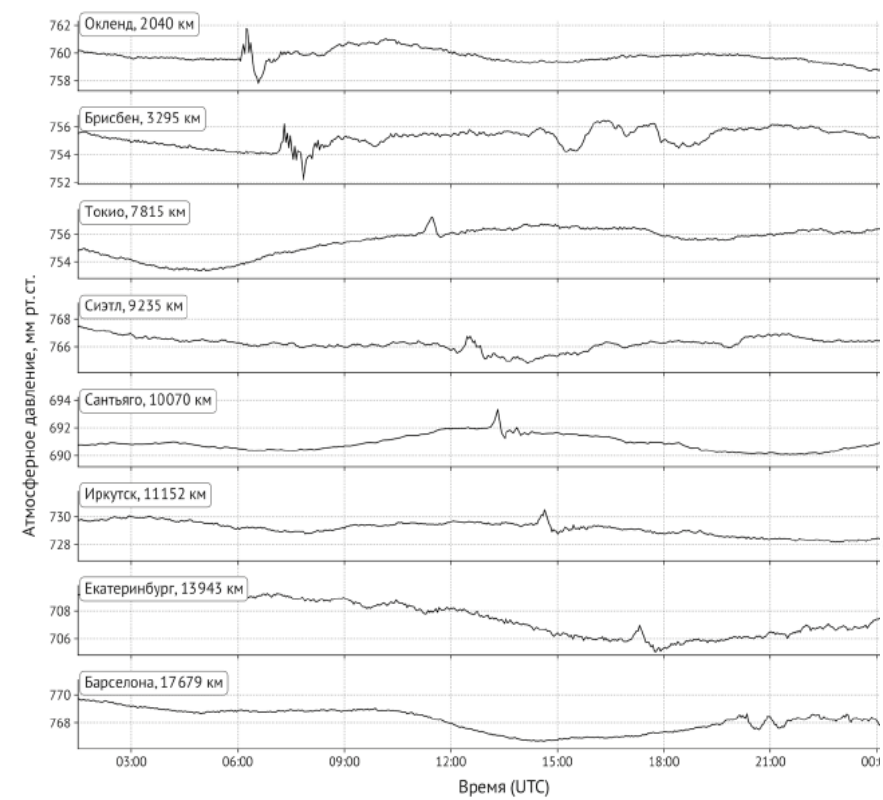
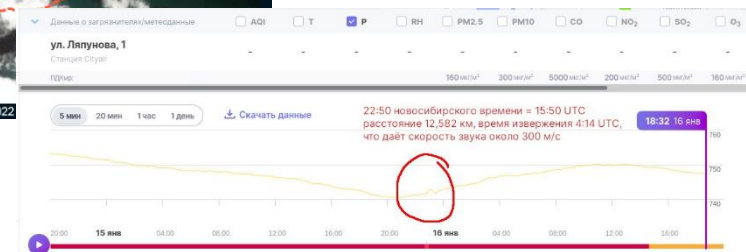
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**V Всероссийская конференция «ВОЛНЫ ЦУНАМИ:  
МОДЕЛИРОВАНИЕ, МОНИТОРИНГ, ПРОГНОЗ» 12.11.2024**

# HungaTonga- Hunga Ha'apai Volcano, 15 Jan. 2022

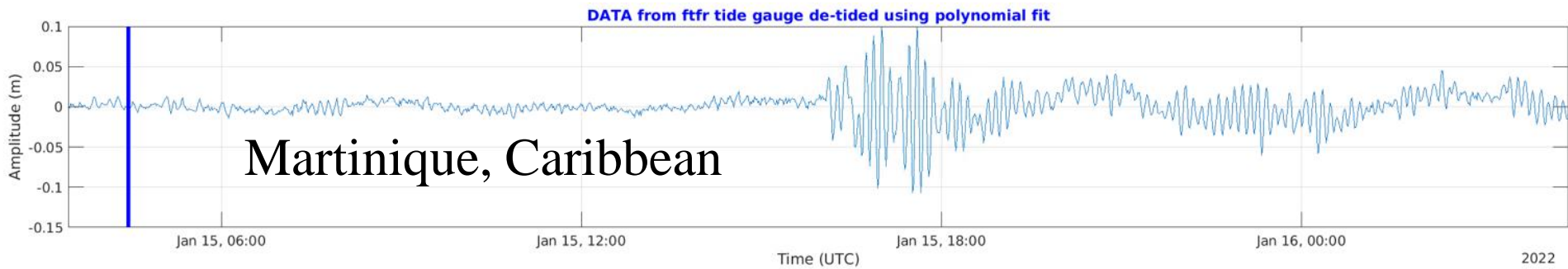
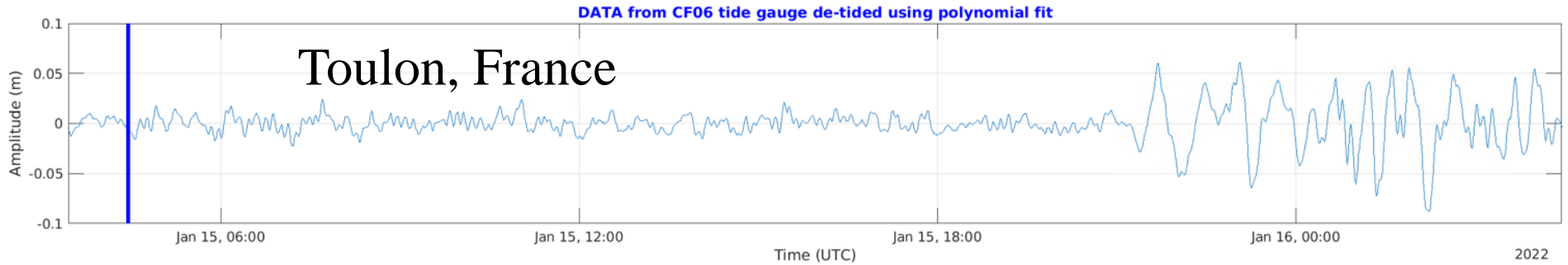
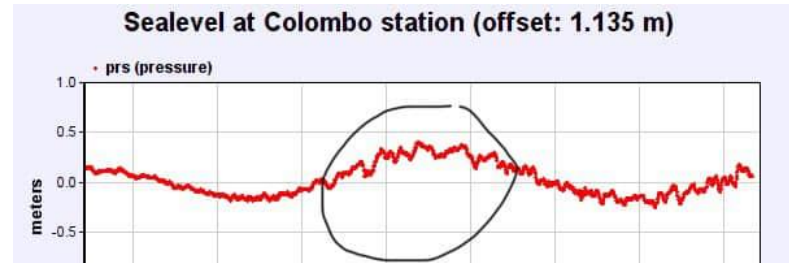
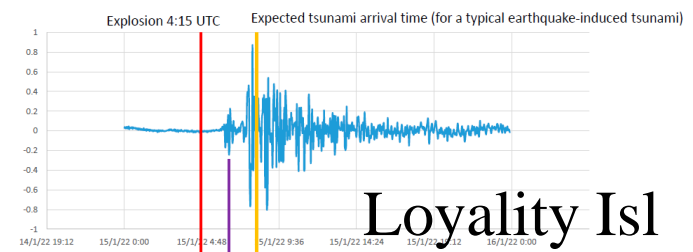
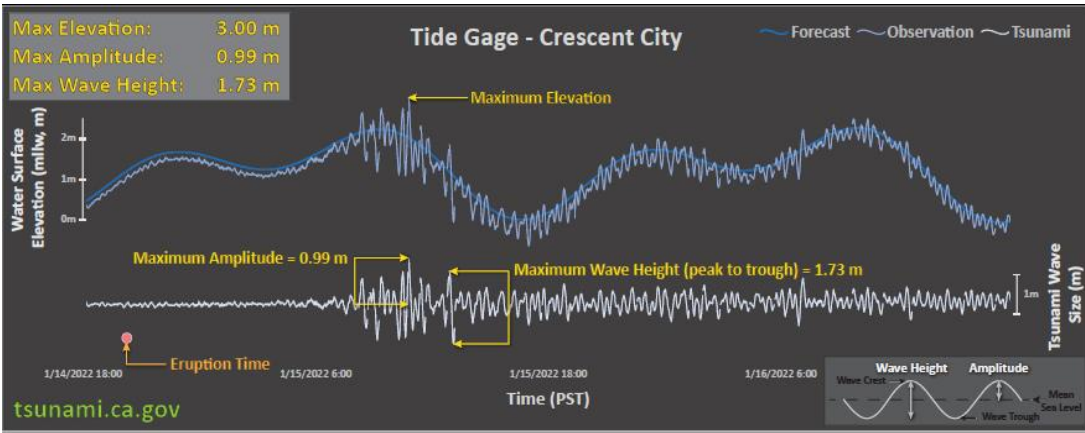


## Sound Waves in Atmosphere





# Tsunami Waves up to 15 m



УДК 551.46

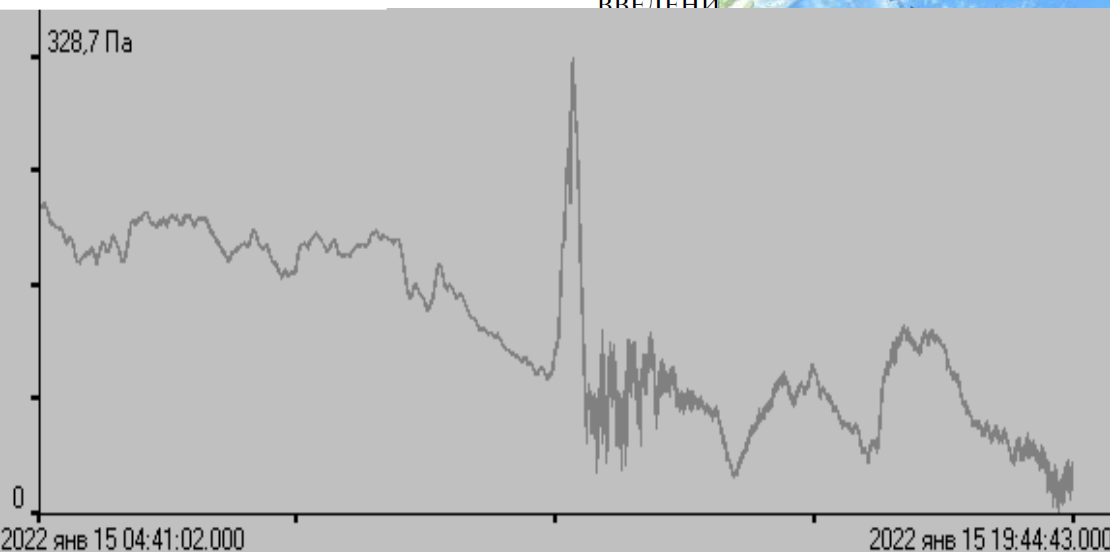
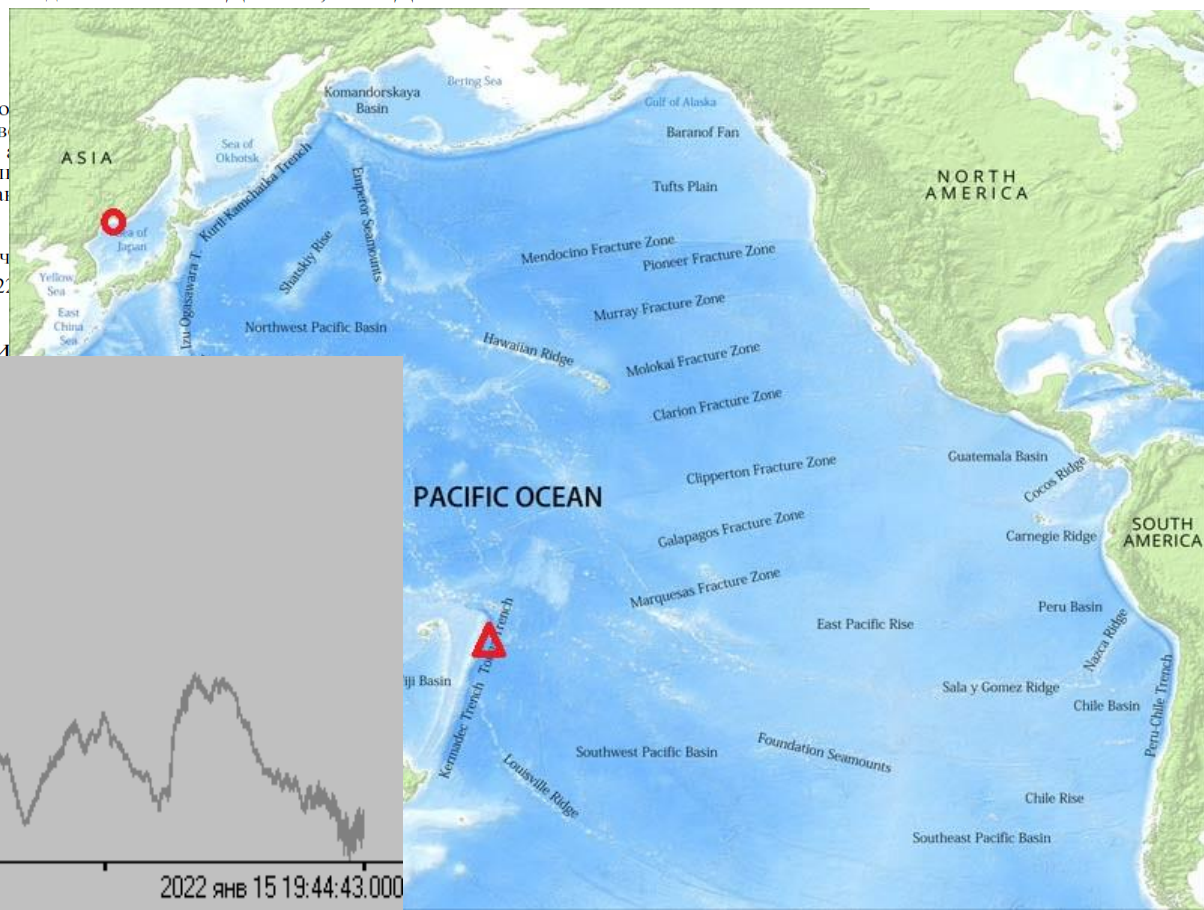
## РЕГИСТРАЦИЯ ВОЗМУЩЕНИЙ В ЯПОНСКОМ МОРЕ, ВЫЗВАННЫХ ИЗВЕРЖЕНИЕМ ВУЛКАНА ХУНГА-ТОНГА-ХААПАЙ В АРХИПЕЛАГЕ ТОНГА 15.01.2022

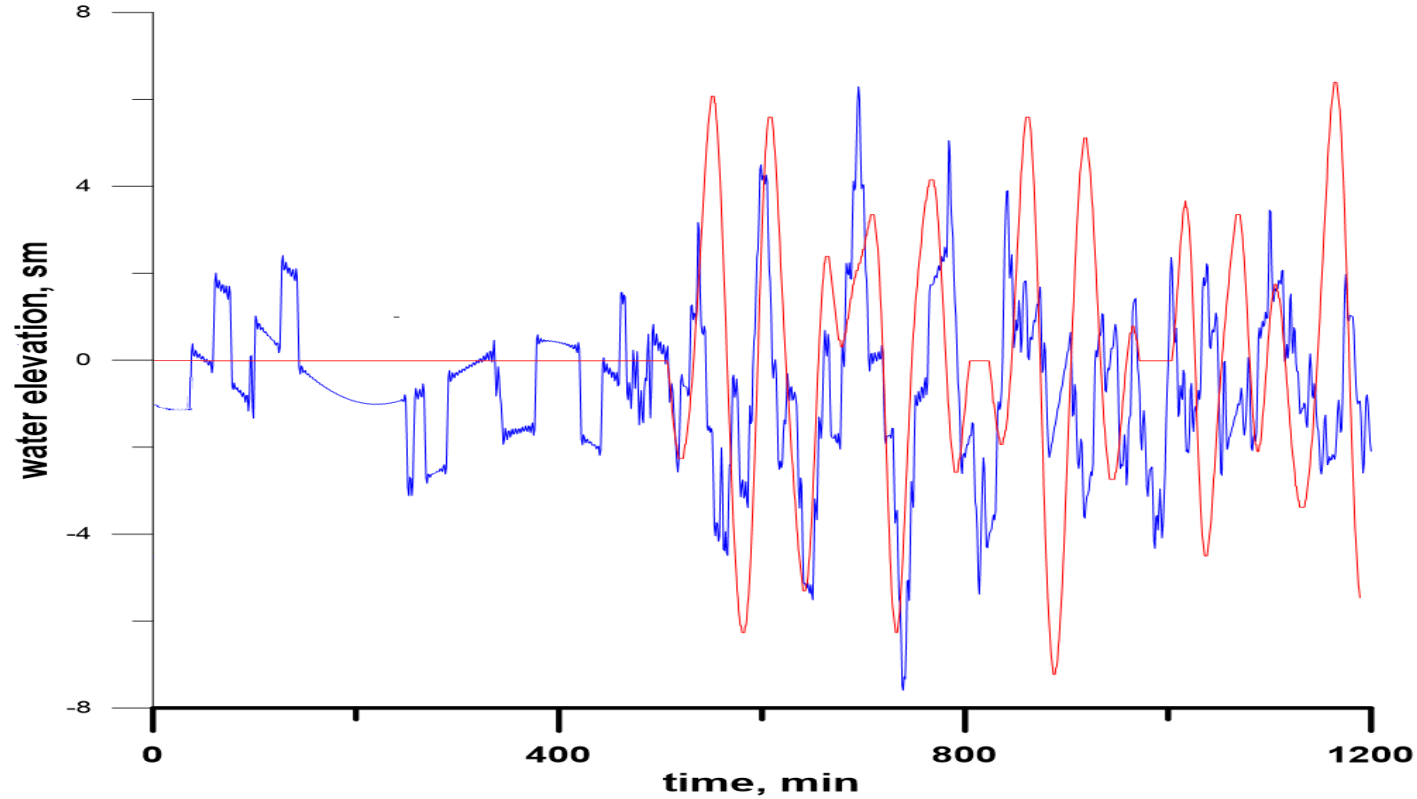
© 2022 г. Член-корреспондент РАН А. И. Зайцев<sup>1,\*</sup>, Е. Н. Пелиновский<sup>2</sup>,  
академик РАН Г. И. Долгих<sup>3</sup>, С. Г. Долгих<sup>3</sup>

15 января 2022 г. случилось извержение вулкана Хунга-Тонга-Хаапай в архипелаге Тонга, сопровождавшееся выбросом пепла на расстояниях от вулкана, колебаниями уровня моря вдали от вулкана и цунами, зафиксированными в Японском море.

**Ключевые слова:** цунами, колебания уровня моря  
**DOI:** 10.31857/S2686739722010001

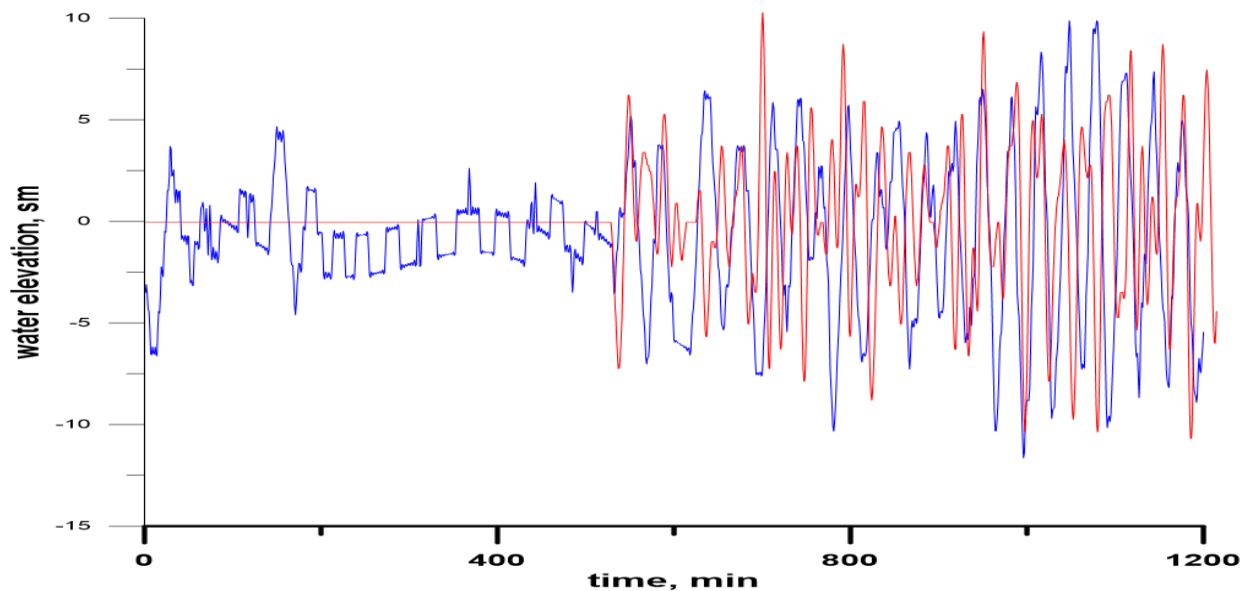
ВВЕДЕНИЕ





**Посьет**

**Владивосток**



**Синяя – наблюдения**

**Красная - расчет**

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## Cover Story

# Oceanic internal waves generated by the Tongan volcano eruption

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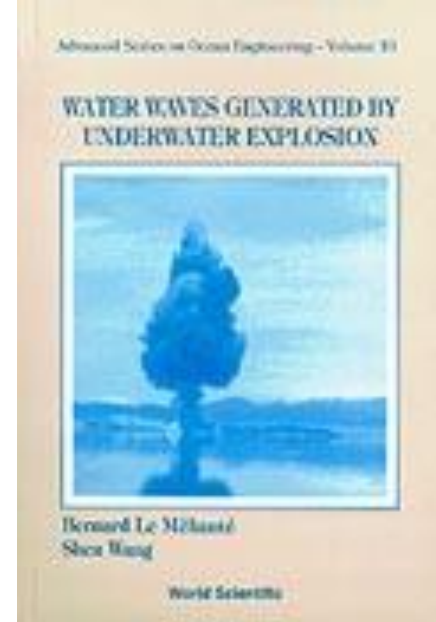
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Internal waves (IW) are widely distributed at the marginal seas or continental shelves (Liu et al., 2013; Zhao and Alford, 2006; Zheng et al., 2007). They have an amplitude of up to hundreds of meters and wave crests of several hundreds of kilometers, and affect ocean environments significantly (Wyatt et al., 2019; Zhang et al., 2022). Satellite images have played an essential role in studying IWs owing to their global-scale observation ability and multi-band sensors in orbit (Alpers, 1985; Apel et al., 1976; Lindsey et al., 2018; Zheng et al., 2001). IW generations are generally reported closely related to wind, tides, topography, and currents (Li et al., 2008; Whalen et al., 2020). Large-amplitude long-wave-crest IW is frequently generated by tide-topography interactions, lee wave mechanism, resonant mechanism, or internal tide steeping in the marginal seas (Xie et al., 2022). Small-scale IW is generated by plume mechanisms or other small-scale disturbances in coastal ocean areas (Alford et al., 2015; Jackson et al., 2012). However, IWs are rarely observed in open ocean areas because of the strong dispersion effect in the deep ocean. Here we report the first observation of IWs generated by a volcano, the Tongan volcano, eruption in the southwest of the Pacific Ocean on January 15, 2022.

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7. Dogan G. et al. Numerical simulations of December 22, 2018 Anak Krakatau tsunami and examination of possible submarine landslide scenarios. *Pure and Applied Geophysics*, 2021, 178, 1-20.
8. Talipova T., et al. Internal tsunami waves in the stratified ocean, induced by explosive volcano eruption: parametric source. *Physics of Fluids* 2024, 36, 4, 042110

# Explosive Volcano Eruption

## Equivalent Source (*Le Mehaute*)

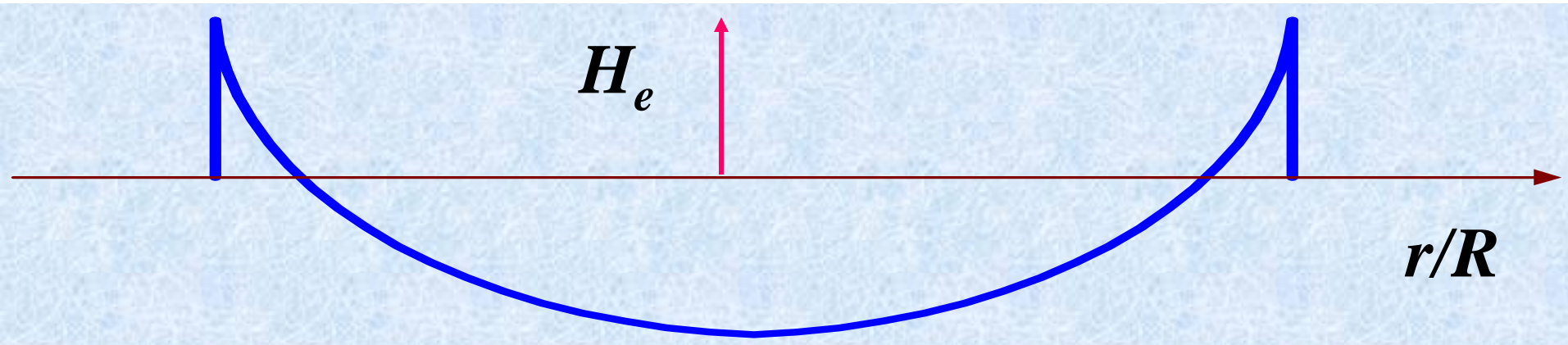


$$H_e \sim E^{0.24}$$

*W* - eruption energy

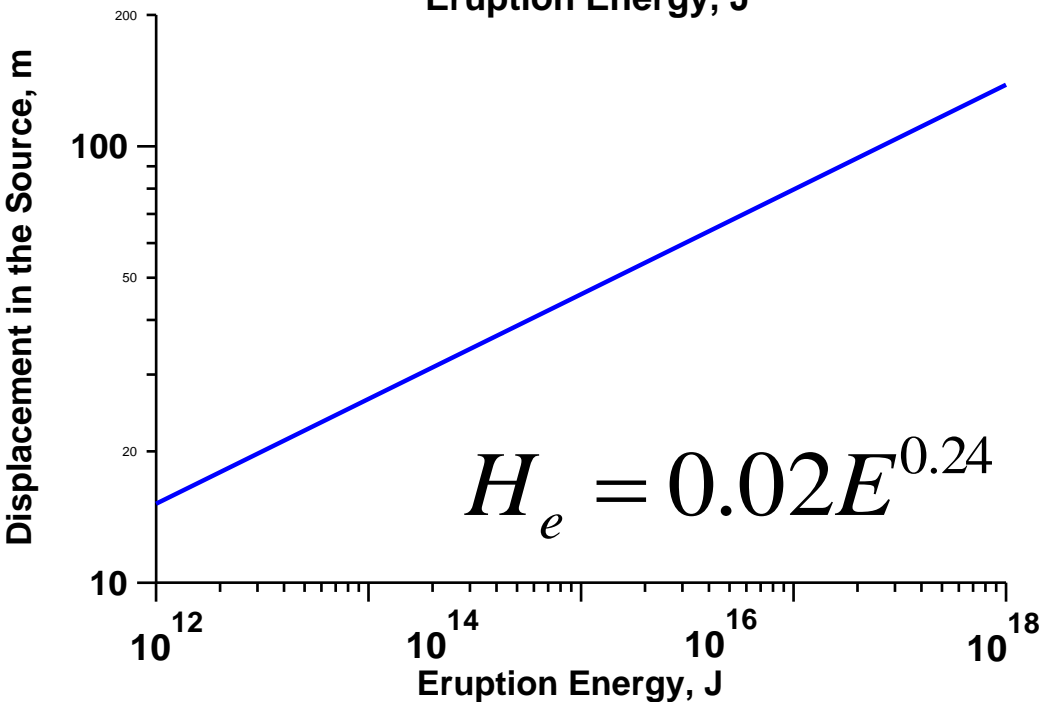
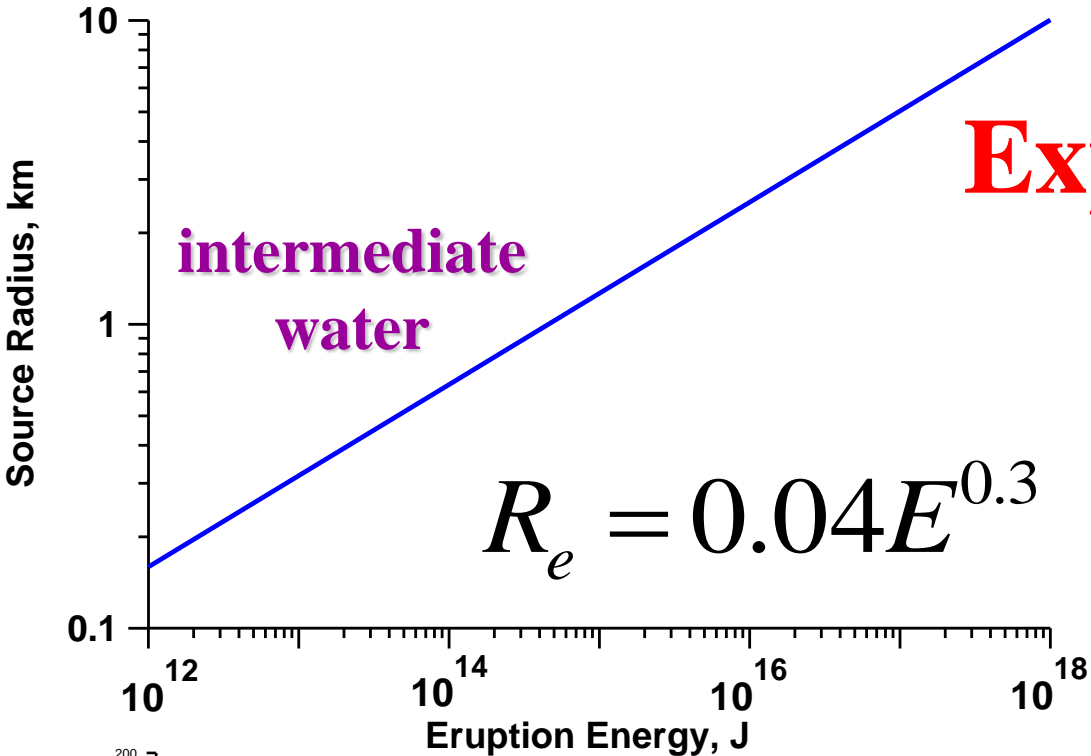
$$R \sim E^{0.3}$$

$$\eta_e(r) = H_e [2(r/R)^2 - 1]$$





# Explosive Tsunamis



**1883**  
**Krakatau eruption**

**$8.4 \times 10^{17}$  Joules**

**$R_e \sim 3.5$  km**

**$H_e \sim 220$  m**

# Linear potential model

$$\Delta\Phi + \frac{\partial^2\Phi}{\partial z^2} = 0$$

**Free surface**

$$\frac{\partial^2\Phi}{\partial t^2} + g \frac{\partial\Phi}{\partial z} = 0$$

**Water Displacement**

**Sea bottom**

$$\frac{\partial\Phi}{\partial z} = 0$$

$$\eta = -\frac{1}{g} \frac{\partial\Phi}{\partial t} (z = 0)$$

# Exact Linear Solution

$$\eta(r, t) = \int_0^{\infty} k dk S(k) J_0(kr) \cos(\omega(k)t)$$

with exact dispersion relation

$$\omega(k) = \sqrt{gk \tanh(kh)}$$

where

$$S(k) = \int_0^{\infty} r dr \eta(r) J_0(kr)$$

# Method of Stationary Phase

$$\eta(r, t) = \int_0^{\infty} k dk S(k) J_0(kr) \cos(\omega(k)t)$$

**for large time and distance**

$$\eta(r, t) \approx \sqrt{\frac{kc_{gr}}{2\pi |dc_{gr}/dk|}} \frac{S(k)}{r} \cos[\omega(k)t - kr - \pi/4]$$

**where**

$$c_{gr}(k) = \frac{d\omega}{dk} = \frac{r}{t}$$



# Variable Amplitude

$$H = \sqrt{\frac{kc_{gr}}{2\pi |dc_{gr}/dk|}} \frac{S(k)}{r} = \frac{Q(k)}{r}$$

**Q(k) has one or several maxima**

$$\mathbf{Max[H] = Max[Q]/r}$$

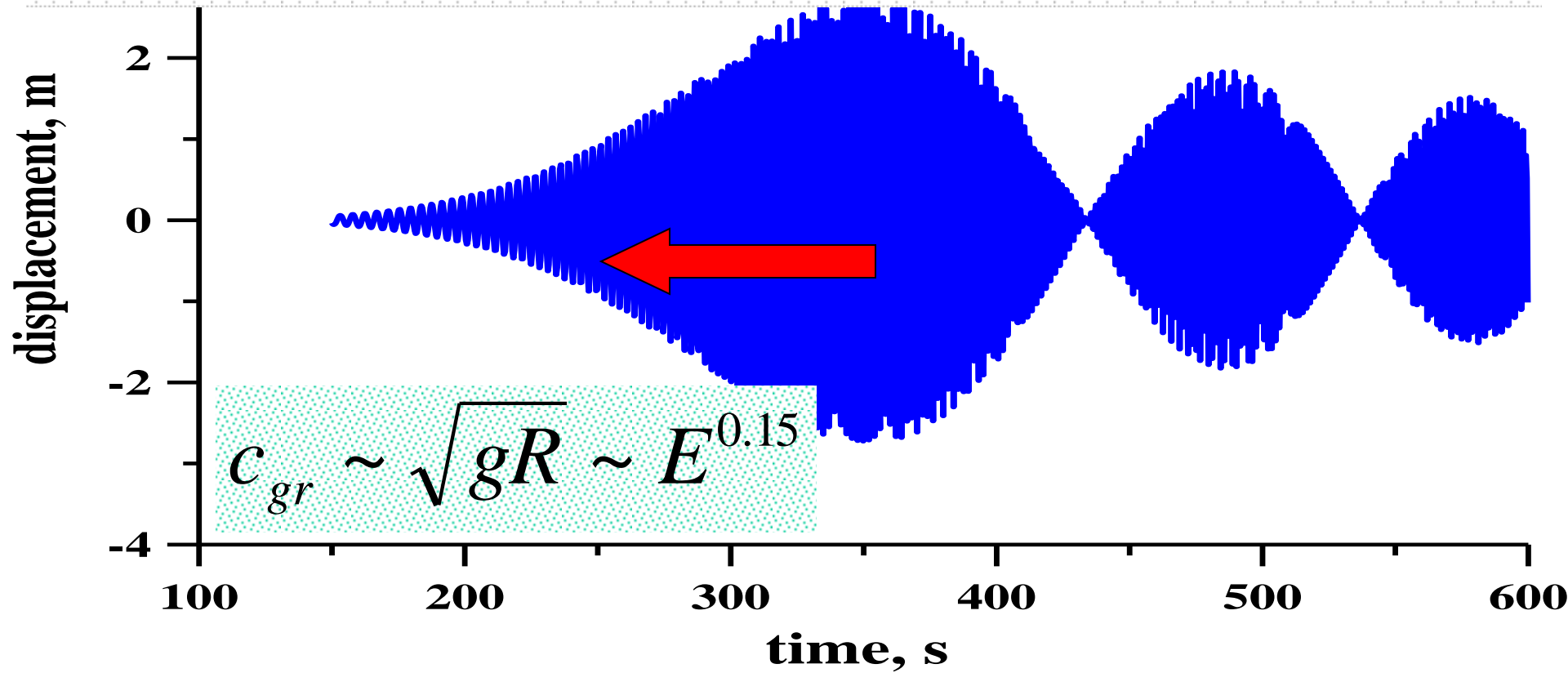
$$c_{gr}(k) = \frac{d\omega}{dk} = \frac{r}{t}$$

**Location of maxima**

# Wave Field far from the Source

(linear deep water)

$$\eta(r, t) \approx \sqrt{2} \frac{H_m R}{r} J_3 \left( \frac{gRt^2}{4r^2} \right) \cdot \cos \left( \frac{gt^2}{4r} \right)$$



# Wave of maximum amplitude

$$H \approx 0.6 H_e \frac{R}{r} \quad \text{height}$$

$$T \approx \pi \sqrt{R / g} \quad \text{period}$$

$$c_{gr} \approx \frac{\sqrt{gR}}{4} \quad \text{speed}$$

# 1952-1953 Myojin-sho volcano

Myōjin-Shō (ja:明神礁 みょうじんしょう) is a submarine volcano located about 450 kilometers south of Tokyo on the Izu-Ogasawara Ridge in the Izu Islands. Volcanic activity has been detected there since 1869.

**Height 0.2-0.9 m Period 70-110 s**

**In tsunami 1% of energy  $E \sim 10^{15}$  J**

**Mirchina N., Pelinovsky E.**

**Estimation of underwater eruption energy based on tsunami wave data.**

***Natural Hazards*, 1988, 1, 277 - 283**

Myōjin-shō



Steam pours from the blocky summit of a lava dome formed at Myōjin-shō during a submarine eruption at the Bayonnaise Rocks volcano in 1952.

<b>Elevation</b>	-50 m (-164 ft)
<b>Location</b>	
<b>Location</b>	Izu Islands, Japan
<b>Coordinates</b>	31°55.1'N 140°1.3'E

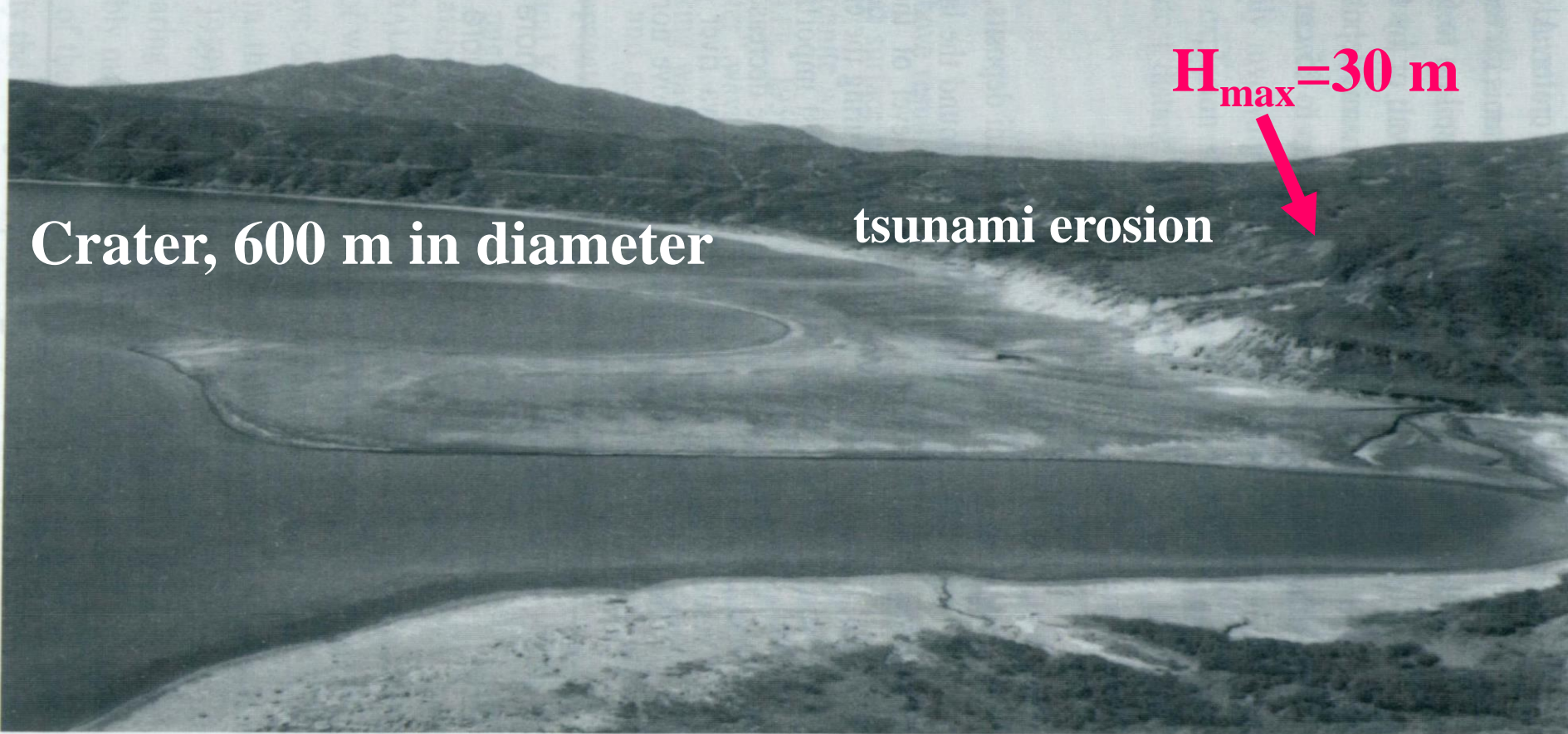
The volcanic eruption from 1952 to 1953 was one of its biggest activities on record, with the repetitious appearance and disappearance of an island, which at one point reached over ten metres above sea level, before sinking after a severe volcanic explosion in September 1953. On September 24, 1953, a survey vessel, *No. 5 Kaiyo-Maru* of the Hydrographic Department of the Maritime Safety Agency, was destroyed by the volcano, with the loss of its crew of 31 (including the nine scientists studying the eruption). Consequently the Department developed *Manbou* (Sunfish), an unmanned radio operating survey boat, and has used it for the research of dangerous sea areas such as submarine volcanoes.



# Tsunami generated by subaquatic volcanic eruptions

*PAGEOPH, 2000, v. 157, 1135-1143*

**January 2, 1996, Karymskoye Lake, Kamchatka, Russia**



**“Explosions occurred every 4 to 12 min. Six explosions were observed with an average interval of 6 min”**

Nat. Hazards Earth Syst. Sci., 10, 2359–2369, 2010

[www.nat-hazards-earth-syst-sci.net/10/2359/2010/](http://www.nat-hazards-earth-syst-sci.net/10/2359/2010/)

doi:10.5194/nhess-10-2359-2010

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**Natural Hazards  
and Earth  
System Sciences**

## **Numerical simulation of a tsunami event during the 1996 volcanic eruption in Karymskoye lake, Kamchatka, Russia**

**T. Torsvik<sup>1</sup>, R. Paris<sup>2</sup>, I. Didenkulova<sup>3,4</sup>, E. Pelinovsky<sup>4</sup>, A. Belousov<sup>5</sup>, and M. Belousova<sup>5,6</sup>**

<sup>1</sup>Bergen Center for Computational Science, Uni Research, Bergen, Norway

<sup>2</sup>CNRS-GEOLAB, Clermont-Université, 4 rue Ledru, 63057 Clermont-Ferrand, France

<sup>3</sup>Laboratory of Wave Engineering, Institute of Cybernetics, Tallinn, Estonia

<sup>4</sup>Department of Nonlinear Geophysical Processes, Institute of Applied Physics, Nizhny Novgorod, Russia

<sup>5</sup>Earth Observatory of Singapore, Nanyang Technological University, Singapore

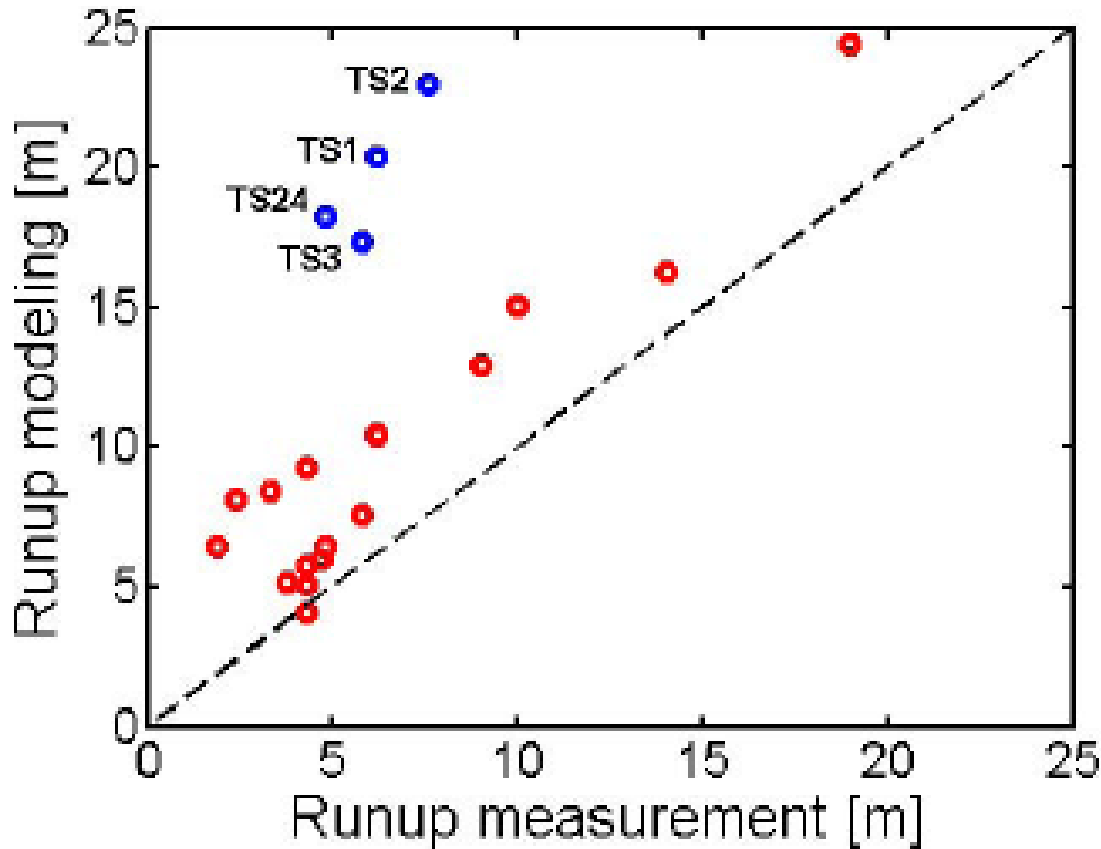
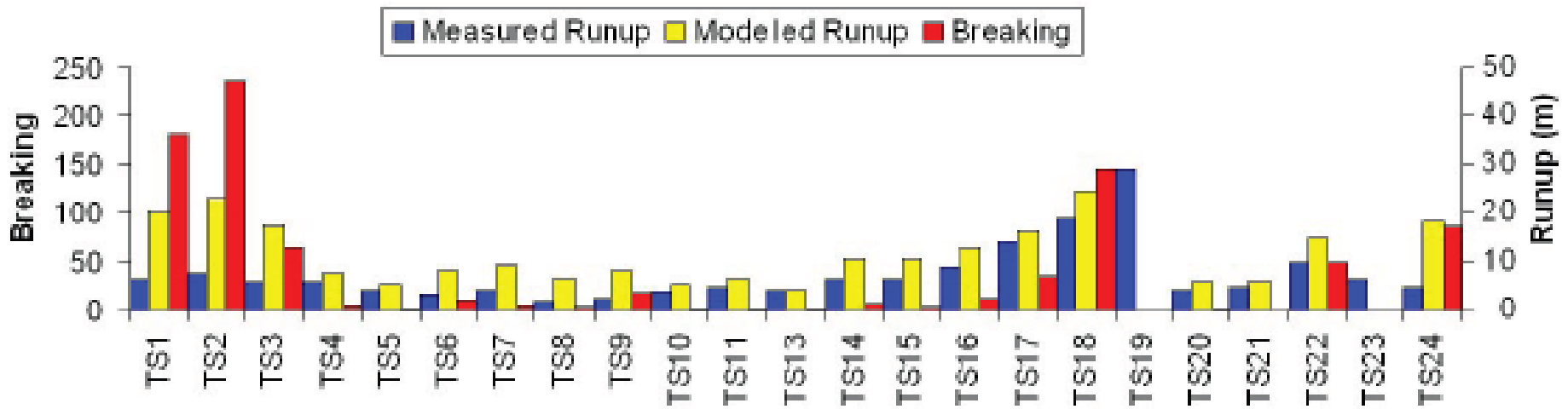
<sup>6</sup>Institute of Volcanology & Seismology, Petropavlovsk-Kamchatsky, Russia

Based on predictions by Belousov and Belousova (2001) that the kinetic energy of the largest explosions were of the order  $E \approx 2 \times 10^{12}$  J, the corresponding maximum initial surface displacement according to Le Mehaute's formula is  $\eta_0 = 23.0$  m. The characteristic length scale  $R = 200.0$  m is determined by the radius of the caldera created by the eruptions.

**Numerical code of nonlinear –dispersive shallow-water theory**  
**COULWAVE**

**Runup - analytical**

$$\frac{R}{H_0} = 2\pi \sqrt{\frac{2L}{\lambda}}$$



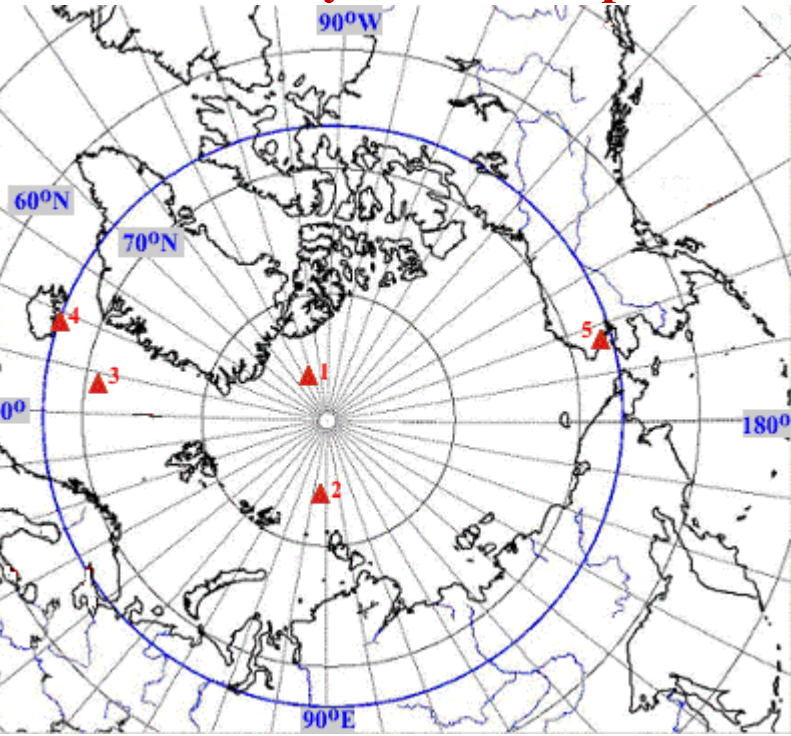
**Very Good Comparison!**



**Ю.И. Блох и др.  
ПОДВОДНЫЙ ВУЛКАН 8.10  
(КУРИЛЬСКАЯ ОСТРОВНАЯ  
ДУГА)**

**ВЕСТНИК КРАУНЦ. НАУКИ О  
ЗЕМЛЕ. 2022. № 3. № 55**

**Подводные вулканы в Арктике**



**и множество  
других...**



# Internal tsunami waves in a stratified ocean induced by explosive volcano eruption: A parametric source

Cite as: Phys. Fluids **36**, 042110 (2024); doi: [10.1063/5.0206121](https://doi.org/10.1063/5.0206121)

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Tatiana Talipova,<sup>1,2</sup> Efim Pelinovsky,<sup>1,3</sup>  and Ekaterina Didenkulova<sup>1,3,a)</sup> 

## AFFILIATIONS

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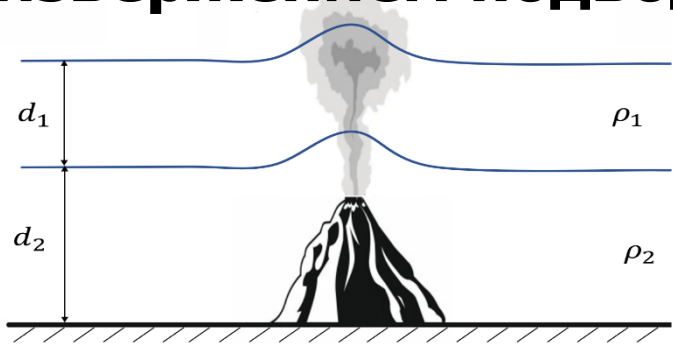
<sup>2</sup>Il'ichev Pacific Oceanological Institute Far Eastern Branch, Russian Academy of Sciences, 43 Baltiyskaya Street, Vladivostok 690041, Russia

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<sup>a)</sup>Author to whom correspondence should be addressed: [edidenkulova@hse.ru](mailto:edidenkulova@hse.ru)

# Open Access

# Внутренние волны цунами, вызванные взрывным извержением подводного вулкана



нормировка:

$$K = kd_1$$

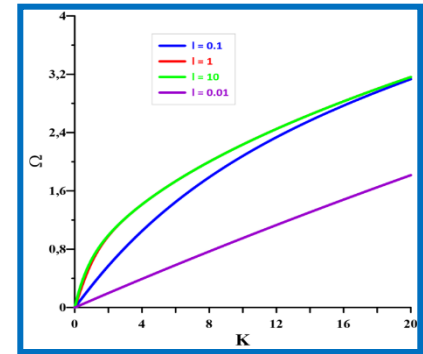
$$\Omega = \omega / N$$

$$N = g' / d_1$$

$$l = \frac{d_2}{d_1}$$

дисперсионное соотношение:

$$\Omega^2(K; l) = \frac{K}{\coth(K) + \coth(Kl)}$$



монохроматическая волна:

$$\eta(x, t) = A \sin(kx - \omega t)$$

дисперсионное соотношение:

групповая скорость:

$$\omega^2 = g' \frac{k}{\coth(kd_1) + \coth(kd_2)}$$

$$g' = g \frac{\Delta\rho}{\rho} = g \frac{\rho_2 - \rho_1}{\rho}$$

$$c_{gr} = \frac{1}{2\Omega} \left\{ \frac{1}{\coth K + \coth(Kl)} + \frac{K}{[\coth K + \coth(Kl)]^2} \left[ \frac{1}{\sinh^2 K} + \frac{l}{\sinh^2(Kl)} \right] \right\}$$

1)  $d_1 = d_2$ :

$$\omega^2 = \frac{1}{2} g' k \tanh(kd)$$

(поверхностные волны)

2)  $d_2$  бесконечен:

$$\omega^2 = g' \frac{k}{1 + \coth(kd_1)} = g' k \frac{\tanh(kd)}{1 + \tanh(kd)}$$

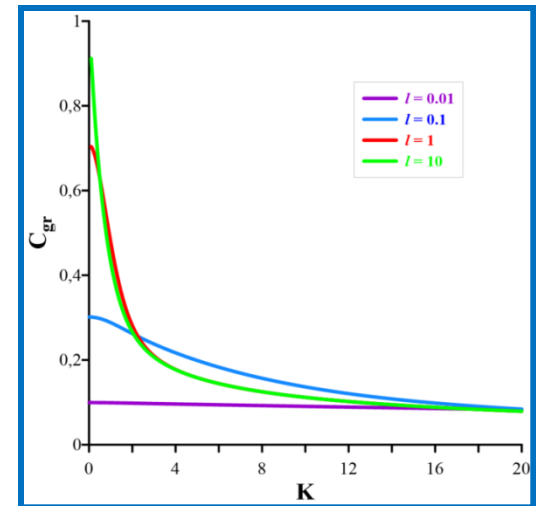
3) оба слоя тонкие:

$$\omega \approx \sqrt{g \frac{d_1 d_2}{d_1 + d_2}} k \left[ 1 - \frac{k^2}{3} \frac{d_1^3 + d_2^3}{d_1 + d_2} \right]$$

(длинные волны)

4) короткие волны:

$$\omega^2 = \frac{1}{2} g' k$$

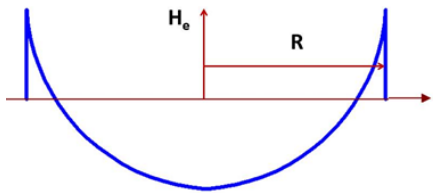


# Внутренние волны цунами, вызванные взрывным извержением подводного вулкана

Решение линейной задачи - решение Кранцера – Келлера  
(преобразование Ханкеля):

Параметрический очаг цунами (параболическая каверна):

$$\eta_e(r) = h \begin{cases} 2\left(\frac{r}{R}\right)^2 - 1 & r < R \\ 0 & r > R \end{cases}$$



$$\eta(r, t) = \int_0^{\infty} k A(k) J_0(kr) \cos(\omega t) dk$$

$$A(k) = \int_0^{\infty} r \eta_e(r) J_0(kr) dr$$

$$\eta(r, t) = hR \int_0^{\infty} J_3(kR) J_0(kr) \cos(\omega t) dk$$

$J_3$  – функция Бесселя третьего порядка

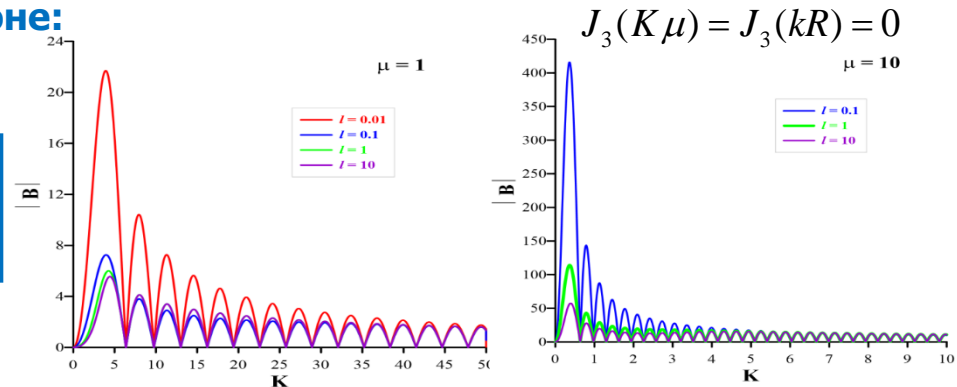
волновое поле разбивается на группы

Характеристики внутренних волн в дальней зоне:

метод стационарной фазы:

$$\eta(r, t) \approx \frac{\eta_e R}{r} \sqrt{\frac{c_{gr}(k)}{k |dc_{gr}/dk|}} J_3(kR) \cos \left[ kr - \omega(k)t - \frac{\pi}{4} \right]$$

$$c_{gr}(k) = \frac{d\omega}{dk} = \frac{r}{t} \quad (\text{поверхностные и внутренние волны})$$

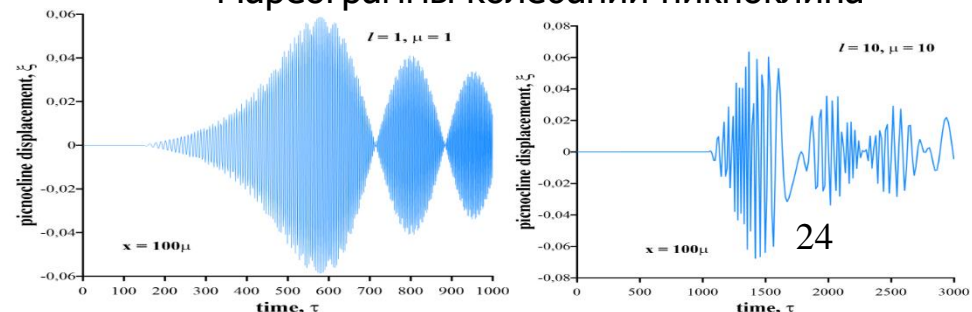


Поле в безразмерных величинах:

$$\xi(x, \tau) \approx \frac{1}{x} B(K; l, \mu) \cos \left[ Kx - \Omega(K)\tau - \frac{\pi}{4} \right]$$

$$c_{gr}(K) = \frac{d\Omega}{dK} = \frac{x}{\tau} \quad B(K; l, \mu) = \mu \sqrt{\frac{c_{gr}(K)}{K |dc_{gr}/dK|}} J_3(K\mu)$$

Мареограммы колебаний пикноклина



# Поле течений на морской поверхности, вызываемое внутренними волнами

Поле горизонтальной скорости течений в верхнем слое:

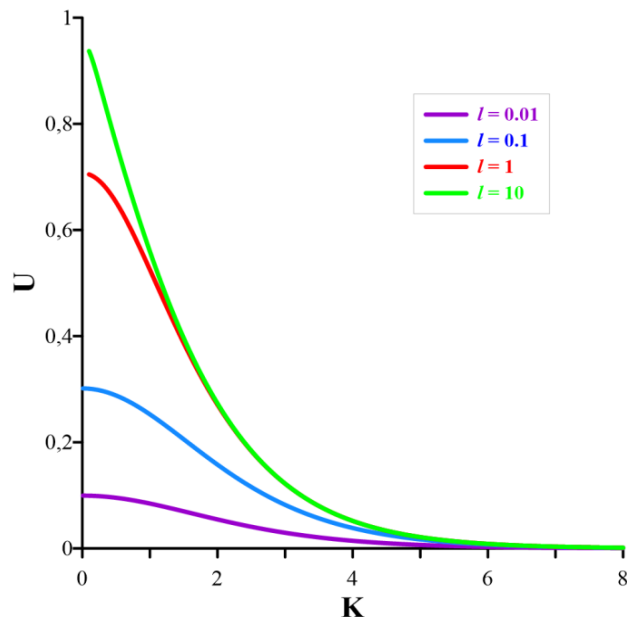
$$u(z, x, t) = \omega A \sin(kx - \omega t) \frac{\cosh(kz)}{\sinh(kd_1)}$$

на морской поверхности:

$$u(z = 0, x, t) = \omega A \sin(kx - \omega t) \frac{1}{\sinh(kd_1)}$$

в безразмерном виде:

$$U(x, t) = \frac{u(z = 0, x, t)}{NA} = \frac{\Omega}{\sinh(K)} \sin(kx - \omega t)$$



скорость течения на морской поверхности в дальней зоне:

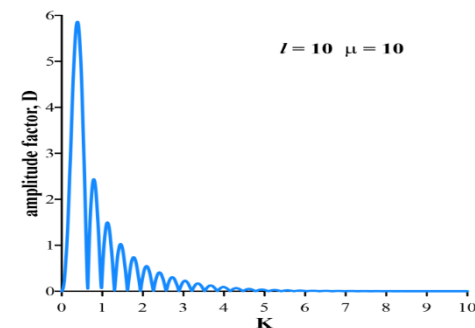
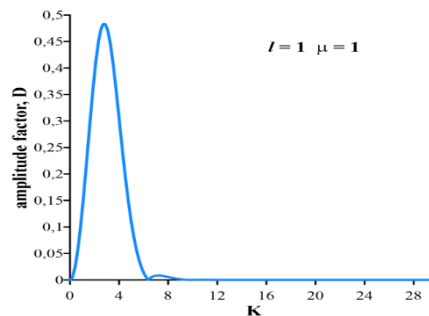
$$u(r, t) \approx \frac{\eta_e R \omega(k)}{r \sinh(kd_1)} \sqrt{\frac{c_{gr}(k)}{k |dc_{gr}/dk|}} J_3(kR) \cos\left[kr - \omega(k)t - \frac{\pi}{4}\right]$$

скорость течения в безразмерном виде:

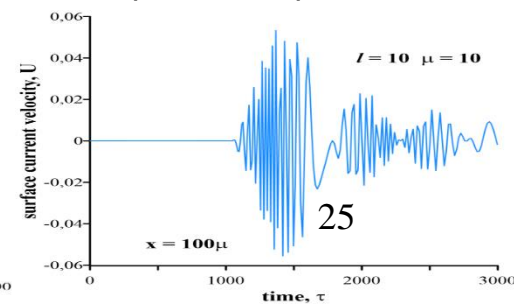
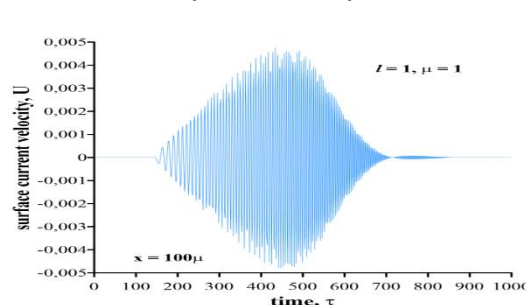
$$u(r, t) = N \eta_e U(x, \tau; \mu, l)$$

$$U(x, \tau) \approx \frac{1}{x} B(K; l, \mu) \frac{\Omega(K)}{\sinh(K)} \cos\left[Kx - \Omega(K)\tau - \frac{\pi}{4}\right]$$

амплитуда вариаций скорости течений,  $D$



Осциллограммы скорости течений на морской поверхности



# Summary:

- ❖ Far-field can be analyzed with equivalent tsunami source
- ❖ Modeling of tsunami source in basic hydrodynamic equations
- ❖ Near-field modeling with strongly nonlinearity
- ❖ Parametrization of other volcano sources: pyroclastic flow

**F. SCHINDELE, L. KONG, E. LANE, R. PARIS, M. RIPEPE, V. TITOV, and R. BAILEY. A Review of Tsunamis Generated by Volcanoes: Source Mechanism, Modelling, Monitoring and Warning Systems. *Pure and Applied Geophysics*, 2024 vol. 181, 1745–1792**